

## An Improved Form for Natural Convection Heat Transfer Correlations

Eugene F. Adiutori  
Ventuno Press  
12887 Valewood Drive  
Naples, Florida 34119  
United States of America  
[efadiutori@aol.com](mailto:efadiutori@aol.com)

---

### Abstract

Natural convection heat transfer correlations are usually in the form  $Nu\{Ra\}$ , and occasionally in the form  $Nu\{Ra\}^*$ . Both forms are inconvenient because they oftentimes require indirect solution:

- When  $Nu\{Ra\}$  correlations are used to calculate heat flux, solution is simple and direct. But when they are used to calculate boundary layer temperature difference, solution must be indirect—i.e. must be based on an indirect method such as iteration or trial-and-error.
- When  $Nu\{Ra\}^*$  correlations are used to calculate boundary layer temperature difference, solution is direct. But when they are used to calculate heat flux, solution must be indirect.

This manuscript describes an improved form for natural convection heat transfer correlations. The improved form allows direct solution for heat flux *and* for boundary layer temperature difference.

Included in this manuscript are graphical and analytical correlations in the improved form obtained by transforming  $Nu\{Ra\}$  correlations from the literature.

---

### Introduction

In forced convection, the heat flux ( $q$ ) is essentially proportional to the boundary layer temperature difference ( $\Delta T$ ). Therefore the heat transfer coefficient ( $h$ ) is a *constant* coefficient—ie its value is independent of  $\Delta T$  and  $q$ . Because  $h$  is a constant coefficient, correlations in the usual form  $Nu\{Re, Pr\}$  can be solved directly for  $q$  *and* for  $\Delta T$ .

In natural convection,  $q$  is a nonlinear function of  $\Delta T$ , and therefore  $h$  is a *variable* coefficient—ie its value is dependent on  $\Delta T$  (or equally on  $q$ ). Because  $h$  is a variable coefficient, correlations in the form  $Nu\{Ra\}$  cannot be solved directly for  $\Delta T$ . They must be solved using an indirect method such as iteration or trial-and-error. Similarly, correlations in the form  $Nu\{Ra\}^*$  cannot be solved directly for  $q$ , but must be solved using an indirect method.

This manuscript describes an improved form for natural convection heat transfer correlations. The improved form allows direct solution for  $q$  *and* for  $\Delta T$ .

### Applications in which $Nu\{Ra\}$ and $Nu\{Ra\}^*$ correlations are solved directly

$Nu\{Ra\}$  correlations are solved directly if the value of  $q$  is to be calculated. The solution is obtained as follows:

- Note that  $Nu\{Ra\}$  is  $(hD/k)\{c_p\rho^2g\beta\Delta TD^3/\mu k\}$ .
- Calculate the value of  $(c_p\rho^2g\beta\Delta TD^3/\mu k)$  from the given information.
- Calculate the value of  $(hD/k)$  from the given  $Nu\{Ra\}$  correlation and the calculated value of  $(c_p\rho^2g\beta\Delta TD^3/\mu k)$ .
- Calculate the value of  $h$  from the calculated value of  $(hD/k)$  and the given information.
- Calculate  $q$  from the calculated value of  $h$ , the given value of  $\Delta T$ , and  $q = h\Delta T$ .

$Nu\{Ra\}^*$  correlations are solved directly if the value of  $\Delta T$  is to be determined. The solution is obtained as follows:

- Note that  $Nu\{Ra\}^*$  is  $(hD/k)\{qc_p\rho^2g\beta D^4/\mu k^2\}$ .
- Calculate the value of  $(qc_p\rho^2g\beta D^4/\mu k^2)$  from the given information.

b	unspecified function of fluid flow rate, fluid properties, and geometry	Nu	Nusselt number $hD/k$ (dimensionless)
Bu	symbol arbitrarily assigned to $qc_p\rho^2g\beta D^4/\mu k^2$ ; identical to NuRa and Ra* (dimensionless)	Pr	Prandtl number $c_p\mu/k$ (dimensionless)
$c_p$	specific heat, J/kgK	q	heat flux, W/m <sup>2</sup>
D	diameter, m	Ra	Rayleigh number $c_p\rho^2g\beta\Delta T D^3/\mu k$ (dimensionless)
G	mass flow rate, kg/s	Ra*	modified Rayleigh number, RaNu (dimensionless)
g	gravity constant, m/s <sup>2</sup>	Re	Reynolds number $DG/\mu$ (dimensionless)
h	heat transfer coefficient, W/m <sup>2</sup> K	T	temperature, K
k	thermal conductivity, W/mK	$\beta$	temperature coefficient of volume expansion, K <sup>-1</sup>
L	length, m	$\mu$	dynamic viscosity, kg/m s
		$\rho$	density, kg/m <sup>3</sup>

- Calculate the value of  $(hD/k)$  from the calculated value of  $\{qc_p\rho^2g\beta D^4/\mu k^2\}$  and the given Nu{Ra\*} correlation.
- Calculate the value of  $h$  from the calculated value of  $(hD/k)$  and the given information.
- Calculate  $\Delta T$  from the calculated value of  $h$ , the given value of  $q$ , and  $\Delta T = q/h$ .

#### Applications in which Nu{Ra} and Nu{Ra\*} correlations cannot be solved directly

Nu{Ra} correlations cannot be solved directly for  $\Delta T$  because neither  $(c_p\rho^2g\beta\Delta T D^3/\mu k)$  nor  $(hD/k)$  can be calculated if  $\Delta T$  is not included in the given information. An indirect solution such as the following is required:

- Note that Nu{Ra} is  $(hD/k)\{c_p\rho^2g\beta\Delta T D^3/\mu k\}$ .
- Select  $\Delta T_1$ , an initial estimate of  $\Delta T$ .
- Calculate  $\{c_p\rho^2g\beta\Delta T_1 D^3/\mu k\}$  from  $\Delta T_1$  and the given information.
- Calculate  $(hD/k)_1$  from the given correlation and the calculated value of  $\{c_p\rho^2g\beta\Delta T_1 D^3/\mu k\}$ .
- Calculate  $h_1$  from  $(hD/k)_1$  and the given information.
- Calculate  $\Delta T_2$  from the calculated value of  $h_1$ , the given value of  $q$ , and Equation (1).

$$\Delta T_2 = q/h_1 \quad (1)$$

- Iterate until convergence is obtained.
- If the solution diverges, select a different iteration scheme. Or select a different indirect method, such as trial-and-error.

Nu{Ra\*} correlations cannot be solved directly for  $q$  because neither  $(hD/k)$  nor  $(qc_p\rho^2g\beta D^4/\mu k^2)$  can be calculated if  $q$  is not included in the given information. An indirect solution analogous to the above is required.

#### The underlying problem with Nu{Ra} and Nu{Ra\*}

The problem with Nu{Ra} may be seen by noting that

$$\text{Nu} \propto h \quad (2)$$

$$\text{Ra} \propto \Delta T \quad (3)$$

Relations (2) and (3) indicate that Nu{Ra} correlations are in the form

$$h = b f(\Delta T) \quad (4)$$

where  $b$  is a function of fluid properties, fluid flow rate, and geometry. The underlying problem with Nu{Ra} correlations is that they are in the form of Eq. (4), a form that allows direct solution for  $h$  if  $\Delta T$  is given, but does not allow direct solution for  $h$  if  $q$  is given.

The problem with Nu{Ra\*} may be seen by noting that

$$\text{Ra}^* \propto q \quad (5)$$

Relations (2) and (5) indicate that Nu{Ra\*} correlations are in the form

$$h = b f\{q\} \quad (6)$$

The underlying problem with Nu{Ra\*} correlations is that they are in the form of Eq. (6), a form that allows direct solution for  $h$  if  $q$  is given, but does not allow direct solution for  $h$  if  $\Delta T$  is given.

#### An improved correlation form that allows direct solution for both $q$ and $\Delta T$

Equation (7) is in a form that allows direct solution for both  $q$  and  $\Delta T$  because the left side is dependent on  $q$  but independent of  $\Delta T$ , and the right side is dependent on  $\Delta T$  but independent of  $q$ .

$$q = b f\{\Delta T\} \quad (7)$$

A dimensionless correlation in the form of Eq. (7) requires the following:

- A dimensionless group that is dependent on  $\Delta T$  and independent of  $q$ .
- A dimensionless group that is dependent on  $q$  and independent of  $\Delta T$ .

Ra satisfies the first requirement. The second requirement is satisfied by the dimensionless group  $qc_p\rho^2g\beta D^4/\mu k^2$ . Let us arbitrarily assign the symbol Bu to this dimensionless group. (Note that Bu is identical to Ra\*, the product of Ra

and  $Nu$ . Also note that  $q\rho^2g\beta D^4/\mu^2k$  also satisfies the second requirement.)

Since  $Bu$  is dependent on  $q$  and independent of  $\Delta T$ , and since  $Ra$  is dependent on  $\Delta T$  and independent of  $q$ ,  $Bu\{Ra\}$  correlations are in the form of Eq. (7).

### Transforming graphical $Nu\{Ra\}$ and $Nu\{Ra^*\}$ correlations to the form of Eq. (7)

Graphical  $Nu\{Ra\}$  correlations are transformed to the form of Eq. (7) in the following manner :

- List the  $Nu, Ra$  coordinates on a spreadsheet.
- Multiply each  $Nu$  coordinate by the corresponding  $Ra$  coordinate to obtain  $Bu, Ra$  coordinates.
- Prepare a  $Bu\{Ra\}$  graphical correlation by plotting the  $Bu, Ra$  coordinates.

Figure 1 is a  $Bu\{Ra\}$  chart obtained by transforming a  $Nu\{Ra\}$  chart that appears in McAdams[1] and also in Kreith and Bohn [2]. Table 1 contains the spreadsheet calculations for the transformation. The  $Nu, Ra$  coordinates in Table 1 are those listed on the  $Nu\{Ra\}$  chart in Kreith and Bohn [2].

Graphical  $Nu\{Ra^*\}$  charts are transformed to the form of Eq. (7) as follows:

- List the  $Nu, Ra^*$  coordinates on a spreadsheet.
- Divide each  $Ra^*$  coordinate by the corresponding  $Nu$  coordinate to obtain  $Ra$  coordinates.
- Multiply each  $Nu$  coordinate by the corresponding  $Ra$  coordinate to obtain  $Bu$  coordinates.
- Prepare a  $Bu\{Ra\}$  graphical correlation by plotting the  $Bu, Ra$  coordinates.

### Transforming analytical $Nu\{Ra\}$ and $Nu\{Ra^*\}$ correlations to dimensionless correlations in the form of Eq. (7)

Analytical  $Nu\{Ra\}$  correlations are transformed to dimensionless correlations in the form of Eq. (7) by multiplying both sides of  $Nu\{Ra\}$  correlations by  $Ra$  or  $Gr$ . Multiplying by  $Ra$  results in dimensionless  $Bu\{Ra\}$  correlations. Table 2 lists several  $Bu\{Ra\}$  correlations obtained by transforming  $Nu\{Ra\}$  correlations from the literature.

Analytical  $Nu\{Ra^*\}$  correlations are transformed to dimensionless correlations in the form of Eq. (7) in the following manner:

- Substitute  $RaNu$  for  $Ra^*$  to obtain  $Nu\{RaNu\}$  correlation.
- Separate  $Ra$  and  $Nu$  to obtain  $Nu\{Ra\}$  correlation.
- Multiply both sides of the  $Nu\{Ra\}$  correlation by  $Ra$  to obtain a  $Bu\{Ra\}$  correlation in the form of Eq. (7).

### Improving the reading precision of $Bu\{Ra\}$ charts

Figure 1 illustrates that  $Bu\{Ra\}$  charts of reasonable size cannot be read with acceptable precision because of the very large range in  $Bu$ . However, the precision of charts of reasonable size can be made acceptable by plotting  $(\log Bu - \log Ra)$  vs  $(\log Bu$  or  $\log Ra)$  as in Figure 2. (Table 1 lists

the calculations that underlie Figure 2.) Figure 2 is read in the following manner:

- From the given information, calculate  $\log Ra$  or  $\log Bu$ . This value establishes the horizontal coordinate.
- If  $\log Ra$  was calculated in the first step, determine  $(\log Bu - \log Ra)$  from the curve marked  $Ra$ . Determine  $\log Bu$  from  $\log Bu = \log Ra + (\log Bu - \log Ra)$ .
- If  $\log Bu$  was calculated in the first step, determine  $(\log Bu - \log Ra)$  from the curve marked  $Bu$ . Determine  $\log Ra$  from  $\log Ra = \log Bu - (\log Bu - \log Ra)$ .

### The impact of film temperature

The fluid properties in natural convection heat transfer correlations are usually evaluated at the film temperature—i.e. at the average temperature in the boundary layer. Since the film temperature usually cannot be determined from the given information, an initial estimate of film temperature must be made, and verified by the subsequent analysis. Thus the use of  $Nu\{Ra\}$  and  $Bu\{Ra\}$  correlations generally involves iteration on the film temperature.

However, the effect is usually so small that the first estimate of film temperature yields a result of sufficient accuracy, and no iteration on film temperature is required.

### Selecting a name for the group $(qc_p\rho^2g\beta D^4/\mu k^2)$

The group  $(qc_p\rho^2g\beta D^4/\mu k^2)$  is often assigned the name “modified Rayleigh number” and the symbol  $Ra^*$ . Since  $Bu$  and  $Ra^*$  are identical,  $Bu\{Ra\}$  correlations are also  $Ra^*\{Ra\}$  correlations. Since  $Ra^*\{Ra\}$  seems poor terminology, it would be desirable to assign a different name and symbol to the group  $(qc_p\rho^2g\beta D^4/\mu k^2)$ .

Lienhard and Lienhard [3] discuss the name and symbol usually assigned to  $(qc_p\rho^2g\beta D^4/\mu k^2)$ :

*To avoid iterating, we need to eliminate  $\Delta T$  from the Rayleigh number. We can do this by introducing a modified Rayleigh number,  $Ra^*$ , defined as  $Ra^* = RaNu$ .*

In the application discussed herein, a modified  $Nu$  is created by multiplying  $Nu$  by  $Ra$ . Based on the reasoning that leads to  $Ra^*$ , the group  $(qc_p\rho^2g\beta D^4/\mu k^2)$  as used herein would be a “modified Nusselt number”, symbol  $Nu^*$ .

It would be misleading to use “modified Nusselt number” and  $Nu^*$  for the group  $(qc_p\rho^2g\beta D^4/\mu k^2)$ . Nusselt number is closely identified with  $h$ , and the group  $(qc_p\rho^2g\beta D^4/\mu k^2)$  does not contain  $h$ .

It therefore seems advisable to assign a new name and symbol to the group  $(qc_p\rho^2g\beta D^4/\mu k^2)$ .

### Conclusions

- $Bu\{Ra\}$  should replace both  $Nu\{Ra\}$  and  $Nu\{Ra^*\}$  because  $Bu\{Ra\}$  allows direct solution for *both*  $q$  and  $\Delta T$ , whereas  $Nu\{Ra\}$  and  $Nu\{Ra^*\}$  do not.

- Graphical and analytical  $Nu\{Ra\}$  and  $Nu\{Ra^*\}$  correlations are readily transformed to  $Bu\{Ra\}$  correlations.
- The group  $(qc_p\rho^2g\beta D^4/\mu k^2)$  should be assigned a new name and a new symbol.

[2] Kreith, F. and Bohn, M.S., (1986), *Principles of Heat Transfer*, p. 251, Harper and Row, New York  
 [3] Lienhard, J.H. IV, and Lienhard, J.H. V, (2003), *A Heat Transfer Textbook*, version 1.21, p. 424, Phlogiston Press, Cambridge  
 [4] Holman, J.P. (1981), *Heat Transfer*, p. 275, McGraw-Hill, New York

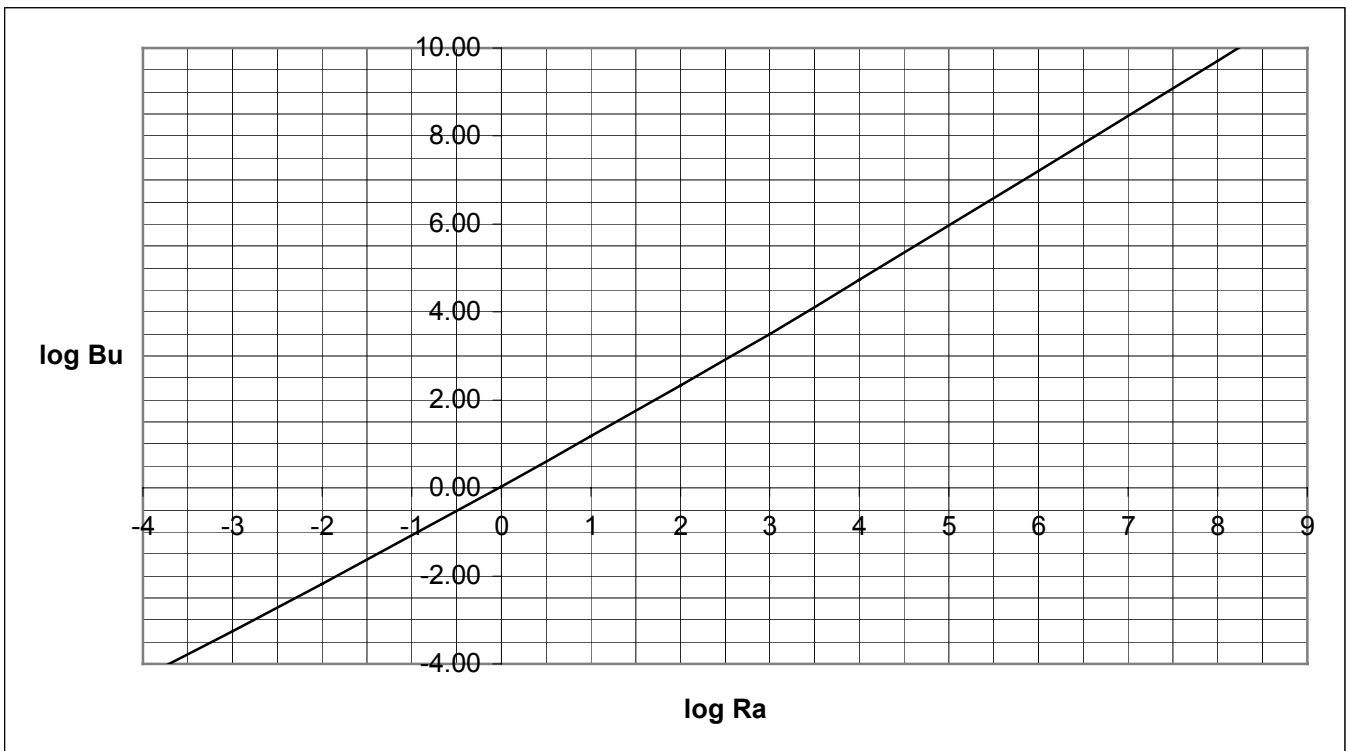
**References**

[1] McAdams, W.H., (1954), *Heat Transmission*, p. 176, McGraw-Hill, New York

**Table 1** Generation of coordinates used in Figures 1 and 2

$Nu^1$	$Ra^1$	$Bu = NuRa$	$\log Ra$	$\log Bu$	$(\log(Bu) - \log(Ra))$
0.49	1.E-04	4.90E-05	-4	-4.31	-0.31
0.55	1.E-03	5.50E-04	-3	-3.26	-0.26
0.66	1.E-02	6.61E-03	-2	-2.18	-0.18
0.84	1.E-01	8.41E-02	-1	-1.08	-0.08
1.08	1.E+00	1.08E+00	0	0.03	0.03
1.51	1.E+01	1.51E+01	1	1.18	0.18
2.11	1.E+02	2.11E+02	2	2.32	0.32
3.16	1.E+03	3.16E+03	3	3.50	0.50
5.37	1.E+04	5.37E+04	4	4.73	0.73
9.33	1.E+05	9.33E+05	5	5.97	0.97
16.2	1.E+06	1.62E+07	6	7.21	1.21
28.8	1.E+07	2.88E+08	7	8.46	1.46
51.3	1.E+08	5.13E+09	8	9.71	1.71
93.3	1.E+09	9.33E+10	9	10.97	1.97

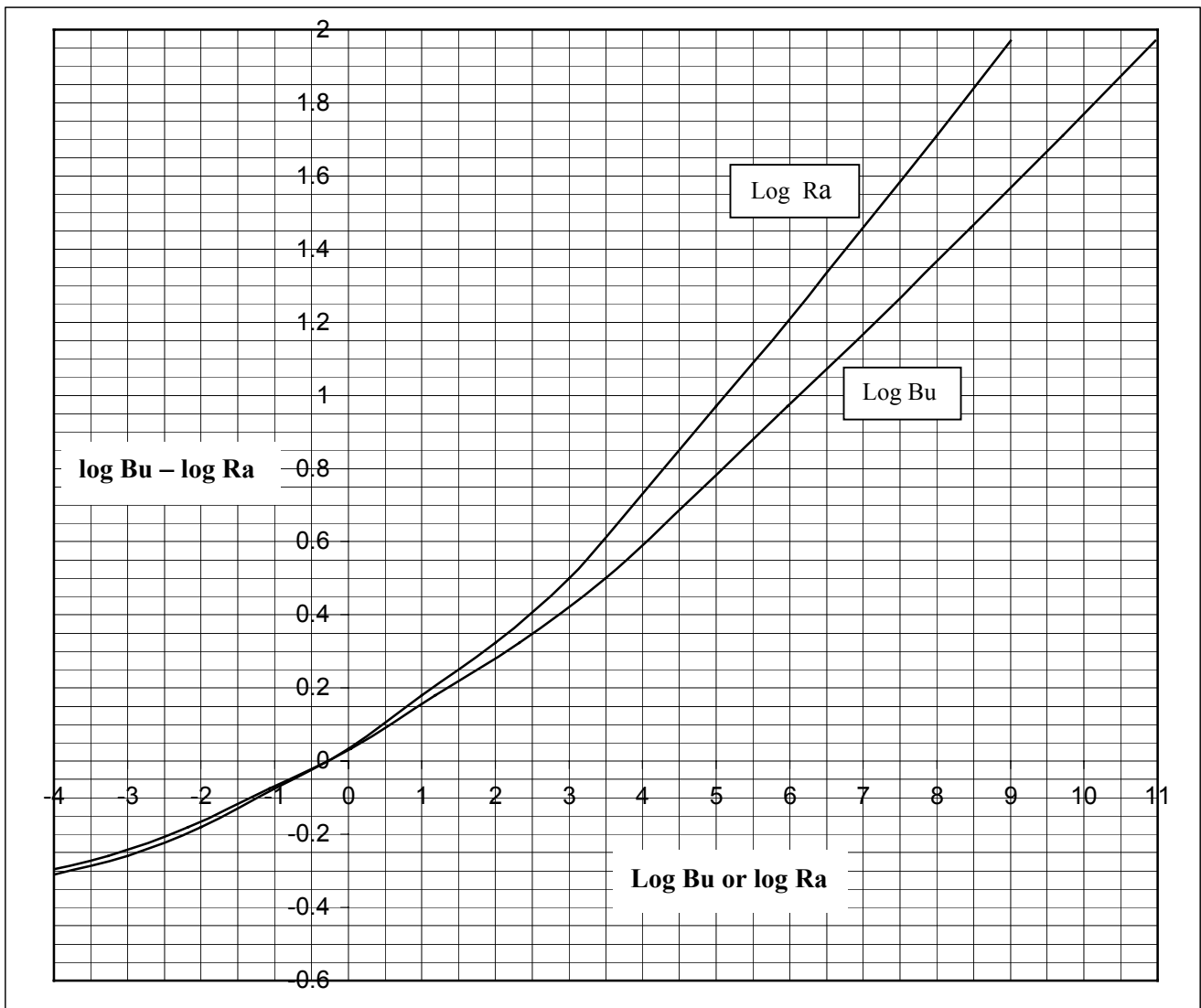
<sup>1</sup> From Kreith & Bohn [2], p. 251



**Figure 1** Natural convection heat transfer from horizontal cylinders

**Table 2** Transformation of literature correlations

Literature Correlation	Dimensionless transformation	Dimensioned transformation
$Nu = .10 Ra^{1/3}$ Holman [4] p 275	$Bu = .10 Ra^{4/3}$	$q = .10 (\Delta T_k/D) Ra^{1/3}$
$Nu = .0210 Ra^{2/5}$ Kreith&Bohn [2] p 253	$Bu = .0210 Ra^{7/5}$	$q = .021(\Delta T_k/L) Ra^{2/5}$
$Nu = .58 Ra^{1/5}$ Lienhard & Lienhard [3] p 423	$Bu = .58 Ra^{6/5}$	$q = .58 (\Delta T_k/L) Ra^{1/5}$



**Figure 2** Natural convection heat transfer from horizontal cylinders

