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### A CRITICAL EXAMINATION OF CORRELATION METHODOLOGY WIDELY USED IN HEAT TRANSFER AND FLUID FLOW

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#### ABSTRACT

The correlation methodology widely used in heat transfer and fluid flow is based on fitting power laws to data. Because all power laws of positive exponent include the point (0,0), this methodology includes the tacit assumption that phenomena are best described by correlations that include the point (0,0).

- If a phenomenon occurs near (0,0), the assumption is obviously valid. For example, laminar flow occurs near (0,0), and therefore the assumption is valid for laminar flow pressure drop correlations.
- If a phenomenon does not occur near (0,0), the assumption is obviously invalid. For example, turbulent flow does not occur near (0,0)—it occurs only after a critical Reynolds number is reached. Therefore the assumption is invalid for turbulent flow pressure drop correlations.

When the assumption is invalid, the correlation methodology widely used in heat transfer and fluid flow is lacking in rigor. The impact of the lack of rigor is evidenced by examples that demonstrate that, when this methodology is applied to phenomena that do not occur in the vicinity of (0,0), highly nonlinear power laws oftentimes result from data that exhibit highly linear behavior.

Because the widely used methodology lacks rigor when applied to phenomena that do not occur near (0,0), power laws based on this methodology are suspect if they purport to describe phenomena that do not occur near (0,0). Data cited in support of such power laws should be recorrelated using rigorous correlation methodology.

Rigorous correlation methodology is also used in heat transfer and fluid flow. It is described in the text, and should become the methodology in general use.

*Keywords: Correlation, methodology, power laws, fluid flow, heat transfer.*

#### INTRODUCTION

The correlation methodology widely used in heat transfer and fluid flow is based on fitting power laws to data. Power laws are equations in the form of Eq. (1):

$$y = mx^n \quad (1)$$

Note in Eq. (1) that if  $x$  equals 0,  $y$  equals 0 for all values of  $m$ , and all positive values of  $n$ . Therefore all power laws of positive exponent necessarily include the point (0,0).

Thus the use of the power law form to correlate data includes the tacit assumption that the observed phenomenon is best described by a correlation that includes the point (0,0). If there is no basis for this assumption (for example, if the phenomenon of interest does not occur near (0,0)), it is lacking in rigor to use a correlation form that necessarily includes (0,0), such as a power law.

Examples of heat transfer and fluid flow phenomena that do not occur near (0,0) are:

- Turbulent fluid flow
- Turbulent heat transfer
- Nucleate boiling heat transfer
- Film boiling heat transfer

Since these phenomena do not occur near (0,0), the widely used correlation methodology is lacking in rigor when applied to them.

The impact of the lack of rigor is evidenced by a review of data cited to support widely accepted, highly nonlinear power law correlations that purport to describe nucleate boiling heat transfer behavior.

The review demonstrates that, when the widely used methodology is applied to phenomena that do not occur near (0,0), highly nonlinear correlations often result from data that exhibit highly linear behavior. Therefore power laws based on the widely used methodology are suspect if they purport to describe the behavior of phenomena that do not occur near (0,0). Data cited in support of such power laws should be re-correlated using rigorous correlation methodology.

Rigorous correlation methodology is also used in heat transfer and fluid flow. It is described in the text, and should become the correlation methodology in general use.

## NOMENCLATURE

- a arbitrary constant
- B arbitrary constant
- m arbitrary constant
- n arbitrary constant
- q numerical value of heat flux in B/hrft<sup>2</sup>
- T numerical value of temperature in degrees F
- ΔT numerical value of boundary layer temperature difference in degrees F
- x unspecified variable
- y unspecified variable

## CORRELATION METHODOLOGY

### Widely Used Methodology

The graphical form of the widely used correlation methodology is described by the following:

- Plot the data (or the values of dimensionless groups determined by dimensional analysis) on log log coordinates.
- Draw a straight line through the data or the dimensionless group results.
- Measure the slope of the straight line and conclude that the data or the dimensionless group results describe a power law in which the exponent equals the slope of the line.

The analytical form of the widely used methodology is described by Cooper [1]:

*Correlations in the form of (power laws) are produced directly from raw data by a . . . least squares program. . . Here the fit is among (logarithms).*

Note that when data are correlated in this way, the power law correlation is determined *directly* from the raw data, and it is not necessary to examine the data, or to plot it.

### The Lack of Rigor in the Widely Used Methodology

It is self-evident that rigorous data correlation includes the following:

- The correlation form is determined by induction—i.e. by examining the data.
- The correlation form places no constraints on the resultant correlation.

The widely used correlation methodology is lacking in rigor because the correlation form is not determined by induction, and because a constraint is placed on the resultant correlation.

- The correlation form is largely determined by the decision to plot the data on log log coordinates, or entirely determined by the decision to analyze the raw data directly using a least squares program where the fit is among logarithms. After the data have been plotted on log log coordinates, or analyzed to determine the fit among logarithms, there is little or no likelihood that the power law correlation form will be rejected.

If the data plotted on log log coordinates exhibit marked curvature, the data may be dismissed as aberrants. Or the curvature may be attributed to a change in regime, and the limits of the regime selected to be sufficiently narrow that a power law does not greatly disagree with the data.

- All power laws of positive exponent include (0,0). Therefore the power law correlation form constrains the resultant correlation to include (0,0).

### Rigorous Methodology

Whether the observed phenomenon does or does not occur near (0,0), rigorous data correlation is achieved in the following manner:

- Plot the data on *linear* coordinates.
- Fair a line through the data points.
- Select correlation forms suggested by the line faired through the data points.
- Quantify the arbitrary constants in the correlation forms so as to optimize agreement between correlation and data.
- Select the correlation that best agrees with the line faired through the data points.

### A Substitute for the Power Law Form

Whenever the power law form is deemed an appropriate correlating form, Eq. (2) should be used in its place.

$$y = mx^n + B \quad (2)$$

If the phenomenon being investigated occurs near (0,0), Eq. (2) should be used in place of the power law form because it quantifies potential bias in the data:

- If optimum correlation is obtained with B significantly different than zero, there is significant bias in the data. The bias is quantified by the value of B. Its impact can oftentimes be eliminated by applying a zero correction to the data (in much the same manner that a zero correction is applied to data from a bathroom scale).
- If optimum correlation is obtained with B not significantly different than zero, bias in the data is not significant.

If the phenomenon being investigated does *not* occur near (0,0), Eq. (2) should be used in place of the power law form in order to determine whether Eq. (2) better correlates the data, and to allow a power law to result from rigorous methodology:

- If optimum correlation is obtained with B significantly different than zero, the data are better correlated by Eq. (2) than a power law.
- If optimum correlation is obtained with B not significantly different than zero, the data are well correlated by a power law. Since Eq. (2) places no constraints on the resultant correlation, the power law is the result of rigorous methodology. (When the power law correlation form is used, the resultant power law is constrained to include (0,0), and therefore it is the result of methodology that is not rigorous.)

## **CORRELATION OF NUCLEATE BOILING $q\{\Delta T\}$ DATA**

### **Onset of Nucleate Boiling**

For more than 100 years, it has been widely recognized that boiling heat transfer does not occur near (0,0). Boiling heat transfer does not occur until a *finite* temperature difference is reached, even if the liquid is saturated. At smaller temperature differences, heat transfer occurs by natural convection, and there is no boiling.

With regard to the onset of boiling, Nukiyama [2] stated:

*In the early stages of my study, I found that the temperature of a metal wire easily reached as high as 105 C without the water boiling. I was in the skies because this was contrary, or so I thought, to the invariable principle that "Water boils at 100C." . . . However, when I happened to read an old textbook, Theory of Heat, written by Clerk Maxwell, Lord Rayleigh, and others, it was lightly described that water boiled when it reached the pertinent boiling temperature for a certain pressure plus the temperature at which the cohesion of the water and its contact surface was overcome, and I realized they had already known the phenomenon.*

### **The Result of Correlating Nucleate Boiling $q\{\Delta T\}$ Data Using Widely Used Methodology**

For more than 50 years, it has been widely accepted that in nucleate boiling, the relationship between  $q$  and  $\Delta T$  is described by power laws in the form of Eq. (3).

$$q = a \Delta T^n \quad (3)$$

where  $n$  is approximately 3. Using the widely used correlation methodology, this result was obtained in the following manner:

1. Plot  $q\{\Delta T\}$  data on a log log chart (or analyze  $q\{\Delta T\}$  data directly using a least squares program where the fit is among logarithms, and omit steps 2 to 4).
2. Draw a straight line through the data.
3. Note that the straight line agrees reasonably well with the data.
4. Note that the slope of the straight line is oftentimes approximately 3.
5. Conclude that nucleate boiling heat transfer exhibits highly nonlinear behavior described by a power law in which the exponent is approximately 3.

McAdams [3] presents 9 log log charts on which straight lines are drawn through nucleate boiling  $q\{\Delta T\}$  data from the literature. The charts validate McAdams' conclusion that:

*While the effect of  $\Delta T$  is significant in all regimes of boiling, it is most important in the range of strong nucleate boiling, for which the data of many observers may be expressed by*

$$q = a_i \Delta T^n \quad (4)$$

where  $n$  is a constant ranging from 3 to 4 . . .

Lienhard and Lienhard [4] present the Rohsenow [5] correlation, a power law in which the  $\Delta T$  exponent equals 3. They also present a log log chart by Rohsenow [5] that demonstrates the correlation agrees well with boiling data from the literature. They state:

*One of the first and most useful correlations for nucleate boiling was that of Rohsenow [5]. . .*

(The  $\Delta T$  exponent in the widely accepted Rohsenow [5] correlation did not result from deduction. It resulted from selection of the exponent that gave optimum agreement with the slope of literature data plotted on log log coordinates.)

The Rohsenow [5] correlation is also presented and recommended by Incropera and Dewitt [6], Eckert and Drake [7], Rohsenow and Hartnett [8], Kreith and Bohn [9], Holman [10], and numerous other texts and articles. Other power law correlations with exponents of 3 to 4 are recommended by McAdams et al [11], Jens and Lottes [12], Levy [13], Kutateladze [14].

It is important to note that nucleate boiling power law correlations are generally validated by demonstrating that straight lines on log log charts agree with the data, and that the slopes of these lines are oftentimes 3 to 4.

### The Result of Correlating Nucleate Boiling $q\{\Delta T\}$ Data Using Rigorous Correlation Methodology

Rigorous correlation methodology is also used in heat transfer and fluid flow. It was used in the following to correlate nucleate boiling  $q\{\Delta T\}$  data: Nukiyama [15]; Mesler and Banchemo [16]; Carne and Charlesworth [17]; Adiutori [18,19]; Ivaskevich et al. [20].

*Without exception*, when nucleate boiling data were correlated using rigorous correlation methodology, it was found that:

- The data describe straight lines on *linear* coordinates.
- The data demonstrate that the value of the  $\Delta T$  exponent is approximately *one*.
- Extrapolation of lines faired through the data generally do not pass through the origin. Therefore the data *deny* correlations that include (0,0).
- The data describe the correlation form of Eq. (5), where  $n$  is essentially and generally equal to one, and  $B$  is dependent on system parameters, and is usually significantly less than zero:

$$q = m\Delta T^n + B \quad (5)$$

Nukiyama is widely regarded as the pioneer of the pool boiling curve. It is surprising that those who followed his lead generally presented boiling data on log log coordinates, even though Nukiyama presented his boiling data on linear coordinates. In the nucleate boiling region, Nukiyama's  $q\{\Delta T\}$  data describe lines that are quite straight. The small degree of curvature exhibited indicates that  $n$  is sometimes slightly less than one, and sometimes slightly greater than one.

A great deal of nucleate boiling data that were initially correlated using the widely used methodology have since been recorrelated using rigorous methodology. Recorrelation has shown that the data exhibit highly linear behavior rather than the highly nonlinear behavior initially described by power laws. The data recorrelated include the data of: Perry [21]; Cichelli and Bonilla [22]; Corty [23]; Stock [24]; Aladiev [25]; and Berenson [26].

Mesler and Banchemo [16] correlated data they obtained, and also literature data, including the data of Cichelli and Bonilla [22], the same data Rohsenow [5] used to validate his power law correlation. They stated:

*(From the data obtained) in this study, it was determined that the nucleate boiling data for organic liquids are well represented by straight lines on a linear plot of heat flux vs. temperature difference. This observation is verified by data in the literature by Cichelli and Bonilla [22], Perry [20], and Corty [23].*

Carne and Charlesworth [17] correlated data they had obtained, and also literature data. They stated:

*Both Berenson [26] and Stock [24] originally presented their data on a log-log basis, but it is evident from Figures 10 and 11 that plotting (their) data arithmetically leads to an equally*

*satisfactory, and in the opinion of the authors of this paper, a better and more meaningful presentation.*

### Berenson's [26] Nucleate Boiling Data Plotted on Linear Coordinates

Berenson [26] obtained very precise boiling data that were presented both graphically and digitally. Berenson's Figure 2 is presented in Appendix 1. It is described by the following:

- The figure is a logarithmic chart on which nucleate, transition, and film boiling data are plotted. The data in the nucleate boiling region are listed in Table 1.
- Straight lines are drawn through the nucleate boiling data. There is good agreement between lines and data points, indicating good correlation with power laws.
- The slopes of the nucleate boiling lines vary from 2 to 5, indicating that the power law exponent is in the range 2 to 5.

Figure 1 herein also presents Berenson's [26] nucleate boiling data listed in Table 1. Figure 1 is a linear chart on which straight lines are drawn through the data points. The good agreement between lines and data points indicates that the data are well correlated by linear equations in the form of Eq. (5) where  $n$  is 1, and  $B$  is dependent on system parameters, and is usually significantly less than zero. The high degree of correlation is evidenced by the fact that the average deviation from the lines is approximately  $1^\circ$  F. Note the following:

- If the correlating form for the data in Figure 1 is determined by induction, a linear correlation will surely be induced.
- The lines in Figure 1 do *not* extrapolate to (0,0). Therefore the data deny correlations that include (0,0).
- Together, Figure 1 herein and Berenson's [26] Figure 2 demonstrate that the correlation methodology in general use readily results in highly nonlinear correlations from data that exhibit highly linear behavior.

### **CONCLUSIONS**

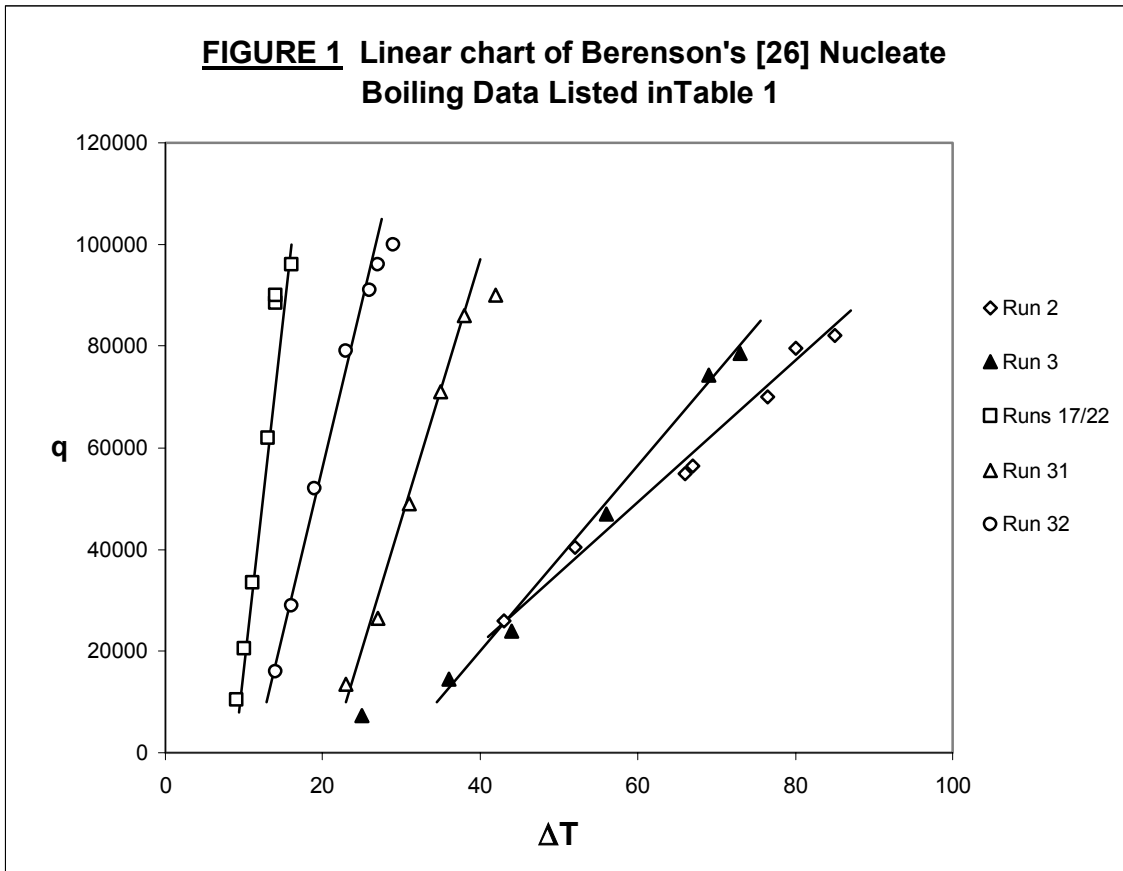
- The correlation methodology widely used in heat transfer and fluid flow is lacking in rigor when applied to phenomena that do not occur near (0,0). The impact of the lack of rigor is evidenced by examples that demonstrate that this methodology oftentimes results in highly nonlinear power law correlations from data that exhibit highly linear behavior.
- Power law correlations based on the widespread correlation methodology are suspect if they purport to describe phenomena that do not occur near (0,0). The data that underlie such correlations should be recorrelated using rigorous correlation methodology.
- Rigorous correlation methodology is also used in heat transfer and fluid flow. It is described in the text, and should become the correlation methodology in general use.

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**TABLE 1** Nucleate Boiling Data Plotted in Figure 2 in Berenson [26] and in Figure 1 Herein

Run 2		Run 3		Runs 17/22		Run 31		Run 32	
$\Delta T$	q	$\Delta T$	q	$\Delta T$	q	$\Delta T$	q	$\Delta T$	q
43	26000	25	7250	9	10500	23	13500	14	16000
52	40500	36	14500	10	20600	27	26500	16	29000
66	55000	44	24000	11	33500	31	49000	19	52000
67	56500	56	47000	13	62000	35	71000	23	79000
76.5	70000	69	74200	14	88500	38	86000	26	91000
80	79500	73	78500	14	90000	42	90000	27	96000
85	82000			16	96000			29	100000



**APPENDIX 1**

Reprinted from *International Journal of Heat and Mass Transfer*, v 5, P. J. Berenson, "Experiments on Pool-Boiling Heat Transfer", Pp 985-999, (1962), with permission from Elsevier.

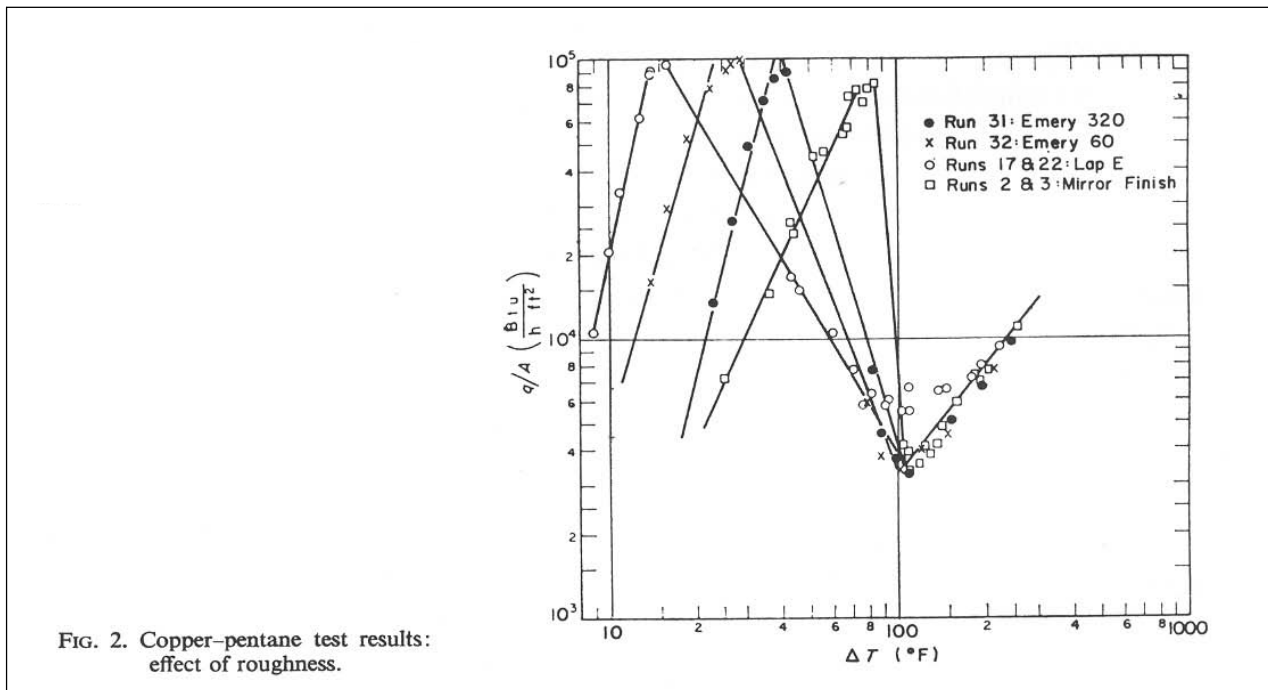


FIG. 2. Copper-pentane test results: effect of roughness.