Fourier—the Father of Modern Engineering

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Joseph Fourier's contributions to modern engineering science are so critically important and so pervasive that he is rightly regarded as the father of modern engineering.

Fourier's contributions, many of which are presented in *The Analytical Theory of Heat* (1822), include:

- The original and still globally accepted view of dimensional homogeneity—the view that natural phenomena can be rigorously described only by equations that are dimensionally homogeneous—ie equations that are dimensionally identical.

  Fourier's view of homogeneity required the creation of contrived parameters such as electrical resistance, heat transfer coefficient, and material modulus. These parameters, and the myriad others like them, are the engineering tools now used to describe and to analyze Natural phenomena. They exist only because of Fourier's pioneering view of homogeneity, and they have all been contrived in the manner pioneered by Fourier.

- The original and still globally used concept of "flux"—of a flow of something per unit area and unit time.

- The original and still globally accepted sciences of convective heat transfer and conductive heat transfer.

- The original and still globally used concepts of heat transfer coefficient and thermal conductivity.

- The original and still globally used solution of "boundary condition" problems by matching the flux at the boundary.

- Many original and still globally used contributions in pure and applied mathematics widely used in modern engineering.

Jean Baptiste Joseph Fourier (1768 - 1830)
Fourier was born in Auxerre, France, the ninth of twelve children. He trained for the priesthood, but spent much of his life teaching mathematics at French universities, principally the Ecole Polytechnique. He was active in the French Revolution, and his activity twice resulted in imprisonment and a potential visit to the guillotine. From 1798 to 1801, he acted as Napoleon's scientific adviser and sometime administrator in the Egyptian campaign. From 1804 to 1807, he

served as prefect of Grenoble, a post he reluctantly accepted because the appointment was made by Napoleon. He was elected to the Academie des Sciences in 1817, and served as its Secretary.

The Analytical Theory of Heat

Grattan-Guinness [1972] describes Fourier's heat transfer publications that eventually resulted in *The Analytical Theory of Heat*:

> *(In 1807, Fourier submitted a paper on heat transfer to the Institute of France.) The paper caused great controversy among the examiners . . . Fourier sent in papers in 1808 and 1809 to meet criticisms, and eventually a prize problem on heat diffusion was proposed by the Institut de France for January 1812, to which he submitted a considerably revised and extended version of the 1807 paper. . . He won the prize; but publication was still delayed. So he began a third version of his work in the form of a book, which eventually appeared as* Theorie analytique de la chaleur *in 1822. The prize paper also appeared—unchanged—in two parts . . . in 1824 and 1826.*

Fourier's view of dimensional homogeneity

Fourier described his view of dimensional homogeneity in the following:

. . . every undetermined magnitude or constant has one dimension proper to itself, and the terms of one and the same equation could not be compared if they had not the same exponent of dimensions. (Article 160)[1].

Fourier did not prove that his then revolutionary view of homogeneity is scientifically rigorous. The only rationale he offered is the following:

> *(This view of homogeneity) is derived from primary notions on quantities; for which reason, in geometry and mechanics, it is the equivalent of the fundamental (axioms) which the Greeks have left us without proof.* (Article 160)

Fourier's contemporaries accepted his revolutionary view of homogeneity, not because he had demonstrated its scientific rigor, but solely because he was able to solve many practical and theoretical problems that had never been solved. He attributed his success to his view of homogeneity. And he attributed the failure of his contemporaries to the lack of homogeneity in their equations, as in the following:

> *If we did not make a complete analysis of the elements of the problem, we should obtain an equation not homogeneous, and, a fortiori, we should not be able to form the equations which express the movement of heat in more complex cases.* (Article 75)

[1] Article numbers locate passages in *The Analytical Theory of Heat*.

Fourier's view of homogeneity expressed in modern terms
White [3] expresses Fourier's view of homogeneity in modern terms:

> *(Dimensional homogeneity) is almost a self-evident axiom in physics. . . (It) can be stated as follows:*
>
> *If an equation truly expresses a proper relationship between variables in a physical process, it will be dimensionally homogeneous; ie each of its additive terms will have the same dimension.*

In other words, scientifically rigorous equations cannot call for the addition or subtraction of terms that have different dimension. For example, ft/sec cannot be added to lbs/ft^3.

The revolutionary nature of Fourier's view of homogeneity
Fourier's view of homogeneity was revolutionary. It *required* the multiplication and division of dissimilar dimensions—mathematical operations that had been deemed *irrational* since Aristotle's time. To satisfy Fourier's view, it was necessary to multiply feet by pounds, to divide pounds by seconds, etc.

Galileo's (1638) view of homogeneity is reflected in the following:

> *If two particles are carried at a uniform rate, the ratio of their speeds will be the product of the ratio of the distances traversed by the inverse ratio of the time-intervals occupied.*

In Galileo's view, distance and time are necessary to quantify speed, but speed has nothing to do with *dividing* distance by time. Galileo divided distance by distance and time by time, but he would *not* divide distance by time. In his view, and in the view of his colleagues, dividing distance by time was irrational.

The view of homogeneity held by Newton and his contemporaries is described by Kroon [1971]:

> *Newton did not concern himself with dimensions or units; he merely expressed proportionality according to the custom of his days.*

The "custom of his days" is reflected in Hooke's law (1676) and in Newton's second law (1686):

> *Stress is proportional to strain.*
>
> *A change in motion is proportional to the motive force impressed . . .*

Note that, in Fourier's view, these expressions are not scientifically rigorous because they are not homogeneous—the dimension of stress is *not* proportional to the dimension of strain—the dimension of motion is *not* proportional to the dimension of force.

That Fourier did in fact conceive the modern view of homogeneity is evidenced by Clerk Maxwell (1891), a scientist of the first rank:

The theory of dimensions was first stated by Fourier, Theorie de Chaleur, Article 160.

Why Fourier's view of homogeneity makes it necessary to contrive parameters
Fourier's view of homogeneity makes it necessary to contrive parameters such as resistances and coefficients because without them, it is not possible to describe engineering phenomena by homogeneous equations. This is illustrated by noting that engineering phenomena are cause-and-effect processes:

- Electromotive force E ***causes*** electric current I.
- Temperature difference ΔT ***causes*** heat flux q.
- Stress σ ***causes*** strain ε.
- Pressure difference ΔP ***causes*** flow rate W.

Because the cause and the effect generally have different dimensions, it is not possible to describe the relationship between the cause and the effect in a homogeneous way using only the cause and the effect parameters. For example, E and I have different dimensions, and therefore it is not possible to describe the relationship between E and I by a homogeneous equation in which only E and I appear. And similarly for ΔT and q, and for σ and ε

How Fourier contrived the parameter "heat transfer coefficient"
Fourier performed a series of heat transfer experiments in order to discover the laws that govern convective heat transfer and conductive heat transfer. He stated:

I have deduced these (heat transfer) laws from prolonged study and attentive comparison of the facts known up to this time: all these facts I have observed afresh in the course of several years with the most exact instruments that have hitherto been used. (Preliminary discourse, p. 2)

Fourier's convective heat transfer data indicated that q is generally proportional to ΔT. Fourier wished to express this result by a law in the form of a homogeneous equation. Expression (1) symbolically describes the observed proportionality between q and ΔT.

$$q \, \alpha \, \Delta T \qquad (1)$$

But in Fourier's view, Expression (1) cannot be the law of convective heat transfer because it is not an equation and it is not homogeneous—the dimension of q is not proportional to the dimension of ΔT.

Equation (2) also accurately describes the observed proportionality between q and ΔT

$$q = a\Delta T \qquad (2)$$

where "a" is a pure number generally referred to as the constant of proportionality. But in Fourier's view, Eq. (2) cannot be the law of convective heat transfer because it is not homogeneous.

In order to describe the relationship between q and ΔT by a law in the form of a homogeneous equation, Fourier used the following rationale to contrive the parameter "heat transfer coefficient":

- Theorize/postulate/assume that the laws of Nature are dimensionally homogeneous. Therefore the law of convective heat transfer must be homogeneous.
- Since the law of convective heat transfer must be homogeneous, the pure number in Eq. (2) must be a parameter that would make Eq. (2) homogeneous.
- State that the pure number in Eq. (2) is in fact a parameter, and this parameter is numerically equal and dimensionally identical to q/ΔT. This transforms the inhomogeneous Eq. (2) to the homogeneous Eq. (3).

$$q = (q/\Delta T)\, \Delta T \qquad (3)$$

- Assign the name "heat transfer coefficient" to the parameter contrived from q/ΔT.
- Assign the symbol "h" to "heat transfer coefficient".

Substituting h for q/ΔT in Eq. (3) results in Eq. (4).

$$q = h\, \Delta T \qquad (4)$$

Equation (4) is the homogeneous law of convective heat transfer conceived by Fourier.[2]

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[2] American heat transfer texts often refer to Eq. (4) as "Newton's law of cooling", and credit Newton with the heat transfer coefficient concept. Newton (1701) is cited as the reference that establishes Newton's priority. Adiutori [1990] states that Fourier, rather than Newton, should be credited with Eq. (4) and the heat transfer coefficient concept. The "law of cooling" presented in Newton (1701) is in fact the inhomogeneous Expression (5).

$$(dT/dt) \, \alpha \, \Delta T \qquad (5)$$

Note that Expression (5) has nothing to do with the concept of flux, whereas Equation (4) explicitly includes the concept of flux, since h is heat *flux* per unit temperature difference. Newton could not have

Note that Fourier contrived the parameter h from the ratio of the cause and effect parameters q and ΔT. Fourier's method of contriving h has been used to contrive other parameters in most branches of engineering:

- State that the ratio of cause and effect is a parameter.
- Assign a name and a unique symbol to the parameter.

The end result of Fourier's view of homogeneity
The end result of Fourier's view of homogeneity is the many contrived parameters used in modern engineering to describe and to analyze engineering phenomena. They are all made necessary by Fourier's view of homogeneity, and they are all contrived in the manner pioneered by Fourier—by combining cause and effect parameters in ratios. For example:

- Electrical resistance is the ratio of the cause and effect parameters electromotive force and electric current.
- Material modulus is the ratio of the cause and effect parameters stress and strain.

In a very real sense, the contrived parameters required by Fourier's view of homogeneity provide the looking glass through which today's engineers view Natural phenomena.

Fourier's view of homogeneity and "electrical resistance"
Georg Ohm performed a series of experiments to discover the law that governs the relationship between E and I. His treatise, *The Galvanic Circuit Investigated Mathematically,* was published in 1827, five years after the publication of Fourier's *The Analytical Theory of Heat.* Ohm expressed his famous law verbally as:

The force of the current in a galvanic circuit is directly as the sum of all the tensions, and inversely as the entire reduced length of the circuit.

Ohm also expressed his law symbolically by the equation

$$I = E/L \qquad (6)$$

where L is the length of an equivalent length of copper wire of a standard diameter. Note that Eq. (6) is obviously *not* homogeneous.

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conceived Eq. (4) or h because he had no understanding of "flux", a concept that did not exist until it was conceived by Fourier almost 100 years after Newton died.

Some time after the publication of Ohm's treatise, his law was transformed to a homogeneous equation in the manner pioneered by Fourier—by stating that the constant of proportionality between E and I is in fact a parameter that is numerically equal and dimensionally identical to the ratio E/I. This parameter was assigned the name "electrical resistance" and the dimension "ohm" (a synonym for volts per ampere). The resultant homogeneous form of the law is

$$E = IR. \tag{7}$$

Maxwell (1873) notes that electrical resistance was in fact contrived in the manner pioneered by Fourier—by combining the cause and effect parameters in a ratio.

> *(Ohm's law states that) the resistance of a conductor . . . is defined to be the ratio of the electromotive force to the strength of the current which it produces.*

The end result is that, because of Fourier's view of homogeneity, electrical engineers today describe and analyze electrical equipment using resistance and resistance theory.

Fourier's view of homogeneity and "material modulus"

Hooke's law is "stress is proportional to strain". Because the dimension of stress differs from the dimension of strain, Hooke's law is inhomogeneous. Therefore it is not acceptable because it does not accord with Fourier's view of homogeneity.

Hooke's law was transformed to a homogeneous equation in the manner pioneered by Fourier—by stating that the proportionality constant between stress and strain is in fact a parameter that is numerically equal and dimensionally identical to the ratio stress/strain. This parameter is now called "material modulus", symbol E. The homogeneous equation form of Hooke's law is called "Young's law".

The end result is that, because of Fourier's view of homogeneity, mechanical engineers today describe and analyze stress/strain behavior using material modulus.

The heat transfer view of Fourier's contemporaries

The magnitude of Fourier's contribution to engineering science in general and to heat transfer engineering in particular is best appreciated by comparing Fourier's understanding of heat transfer phenomena with that of his contemporaries.

Herivel (1975) describes the view of convective heat transfer held by Laplace and Biot, and compares it with Fourier's view:

> *Laplace's (and presumably Biot's) views on the question (of convective heat transfer) are given at the end of the section on heat in Laplace's 1809 paper on diffraction. There he assumes that the surface of a heated body rapidly reaches that of the*

surrounding medium, and that a law is then quickly established governing the rise of temperature within the body up to a certain maximum value U. The loss of heat is then proportional to U. This is opposed to the views of those (including Fourier and Newton!) who thought that the temperature of the surface was above that of the surrounding medium, thus breaking the law of continuity. . . The rather wild nature of Laplace's hypothesis in this matter is in striking contrast with the sober, simple, and correct condition formulated by Fourier, and provides another example of his superior physical intuition in this particular topic compared with that of Laplace, Poisson, or Biot.

Note that *"Laplace's (and presumably Biot's) views"* were based on the erroneous assumption that temperature is *continuous* at the surface of a heated body—ie they assumed that the heat transfer coefficient at the surface of any warm body is infinite!!

Biot (1804) published a paper on the temperature distribution in iron bars heated at one end. Based on testing a single iron bar, Biot concluded:

Thus it is physically impossible to heat to one degree the end of an iron bar of two metres in length by heating the other end, because it would melt before this.

Fourier referred to Biot's conclusion in the following:

(Equations that describe the temperature distribution in a heated bar must include) the dimensions of the (bar), in order that we might not regard as general, consequences which observation furnished in a particular case. Thus, it was discovered by experiment that a bar of iron, heated at one extremity, could not acquire, at a distance of six feet from the source, a temperature of one degree; for to produce this effect, it would be necessary for the heat of the source to surpass considerably the point of fusion of iron; (Article 75)

Fourier confidently corrects Biot's conclusion in the following:

. . . this result depends on the thickness of the prism employed. If it had been greater, the heat would have been propagated to a greater distance, that is to say, the point of the bar which acquires a fixed temperature of one degree is much more remote from the source when the bar is thicker, all other conditions remaining the same. We can always raise by one degree the temperature of one end of a bar of iron, by heating the solid at the other end; we need only give the radius of the base a sufficient length; which is, we may say, evident, and of which besides a proof will be found in . . . (Article 78). Article 76

Article 78 presents the equation that *quantitatively* describes the temperature distribution in a bar heated at one end. Article 80 presents the equation that *quantitatively* describes the quantity of heat flowing through a section of the bar as a function of distance from the heated end.

In summary, the heat transfer understanding of Fourier's contemporaries is charitably described as groping in the dark, whereas Fourier's understanding is accurately described as a quantitative and comprehensive science of heat transfer.

Fourier's concept of flux, and the resistance it met from his contemporaries
Fourier's concept of "flux" is critically important in most branches of engineering. With regard to the originality of this concept and the resistance it met from Fourier's contemporaries, Herivel states:

> *The notion of a flux of heat or other "substance" as a rate of flow per unit time per unit area is such a familiar and central one in modern theoretical physics, that it is difficult if not impossible to assess the measure of originality involved in its original formulation by consideration of the concept itself. . . Fourier's contemporaries . . . found it excessively difficult either to understand or to accept this concept. Thus Laplace, by any reckoning the foremost theoretical physicist among Fourier's contemporaries in France or elsewhere, certainly did not understand this basic element of the analytical theory of heat when he first encountered it as a member of the commission set up to report on the 1807 memoir, and the criticism of the derivation of the basic equation in the report on Fourier's Prize Essay proves that he still had not accepted it by February 1812. Biot and Poisson were even more obtuse than Laplace. As late as 1816 they were still insisting on the existence of an "analytical difficulty" . . . (in Fourier's) notion of heat flux.*
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> *. . . five years elapsed between the time that Fourier first entertained the notion of heat flux, and the absolutely clear exposition found in (his letter of 1810). Given the difficulty which Fourier himself experienced in clarifying this concept, Laplace, Biot, and Poisson must not be judged too harshly for their failure to welcome it with open arms . . . All in all this is surely another example of one of those apparently simple, almost trivial, concepts in theoretical physics which nevertheless seem to require for their formulation the intervention of a Galileo or a Newton.*

Fourier's contributions in pure and applied mathematics
Lienhard [1983] summarizes Fourier's contributions in pure and applied mathematics presented in his 1807 memoir on heat transfer:

> *Fourier submitted a new 234 page manuscript to the Institut de France in Paris in 1807. In it he did something more important than determining how to formulate the laws governing the flow of heat in a solid. He did something beyond updating Bernoulli's trigonometric series to solve the equation. He actually provided us with the strategies that would be basic to the entire field of continuum mechanics, of which heat conduction and convection are a major part. These are the identification of field differential equations and boundary conditions, the technique of separation of*

variables, and the idea of representing solutions in the form of series of arbitrary functions.

The reason for the long delay between presentation and publication
Herivel describes the reason Fourier's 1807 paper was never published by the Institute of France, and why his prize winning paper of 1812 was not published by the Institute until 1824 (Part 1) and 1826 (Part 2).

> *. . . The controversy arising out of the 1807 memoir (seriously jeopardized the acceptance of Fourier's work). Laplace and Lagrange remained openly opposed to it and Biot lost no opportunity of sniping at it from the sidelines. . . To ward off the attacks of Biot and Laplace, to neutralize if not remove the misgivings of Lagrange, required the protracted exercise of all Fourier's considerable powers of persuasion . . . The persuasive eloquence which had pleaded the case of the innocent before the popular tribunes in Auxerre during the (French) Revolution . . . was now pressed into service to defend a theory which was fighting for its life against the conspiracy of Biot, Laplace, and Poisson. . .*
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> *. . . if (Fourier) had not reacted vigorously, and on occasion ruthlessly, against the Biot-Laplace conspiracy, if he had not had the daring and effrontery to criticize Laplace openly . . . then there is every reason to believe that Fourier's paper would have been forgotten, the subject of the propagation of heat would not have been set as a Prize Essay, or that if it had, Fourier would have been too discouraged by the reception of his earlier memoir to submit another.*

Fourier, the father of modern engineering
Joseph Fourier's many contributions to modern engineering science are so critically important and so pervasive that he is rightly regarded as the father of modern engineering.

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