Thermal Stability of Surfaces Heated by Convection and Cooled by Boiling

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In past discussions of the thermal stability of a wall heated by convection and cooled by boiling, the thermal capacitance of the wall has been lumped at a single point in the wall and the thermal resistance of the wall has usually been neglected. A more complete analysis, considering the distribution of resistance and capacitance through the wall, shows that the lumped analysis may lead to spurious or erroneous conclusions and that the wall resistance has a destabilizing effect on the system. Specifically, for a flat wall stability requires that $-M < 1/[(x_2 - x_1/k) + (1/h)]$. For a hollow circular cylinder with boiling on the outside, $-k/Mr_2 > k/h_1 + \ln(r_2/r_1)$, and with boiling on the inside, $-k/Mr_1 > k/hr_2 + \ln(r_2/r_1)$. M is the slope of the boiling curve, h is the convective heat transfer coefficient, $(x_2 - x_1)$ is the thickness of the flat wall, r_1 and r_2 are the inside and outside radii of the cylinder, respectively, and k is the thermal conductivity of the wall.

In the design of heat transfer equipment it is desirable that a small change in any single operating parameter produce a corresponding small change in the other system parameters; *i.e.*, it is desirable that the system possess the characteristic called "stability." The condition for the thermal stability of a heated wall, as first presented by Adiutori (1964), may be expressed as

$$\frac{dq_i}{dT_w} \leq \frac{dq_o}{dT_w} \tag{1}$$

where q_o is the rate of heat removal from the wall, q_i is the rate of heat addition to the wall, and T_w is the temperature of the wall. This condition is satisfied in most thermal designs since the left-hand term is usually negative and the right-hand term is usually positive. The one notable exception occurs in the design of devices cooled by boiling. In the transition region of boiling the heat flux decreases with increasing surface temperature and, as a consequence, this region is unstable under many ordinary design conditions.

To apply eq 1 to a specific thermal design problem involving boiling and to predict the system behavior under unstable conditions it is necessary that the slope of the boiling curve be known in the transition region. Wallis and Collier (1967) applied eq 1 to several different design situations, and their results indicate that a convectively heated wall might be used to obtain data in the transition region. As normally presented, the heat input to the wall is

$$q_i = h(T_f - T_w) \tag{2}$$

where h is the convective heat transfer coefficient between the hot fluid, at temperature T_f, and the wall, at temperature T_w. Then

$$\frac{dq_i}{dT_w} = -h \tag{3}$$

and

$$\frac{dq_o}{dT_w} = M \tag{4}$$

where M is the local value of the slope of the boiling curve where $q_o = q_i$, *i.e.*, under equilibrium conditions. The stability criterion is, from eq 1

$$h \geq -M \tag{5}$$

In the transition region near the peak heat flux M is negative and is apparently quite large. Stability, then, requires a very large value of h.

The crucial point in the preceding analysis is that eq 1 is valid only if all of the thermal capacitance of the wall is lumped at T_w; *i.e.*, it is valid only if the thermal resistance of the wall is neglected. A more rigorous analysis must include the wall resistance and must treat the capacitance as a distributed parameter.

Analysis

Consider the general problem of one-dimensional heat conduction through a shell of constant thickness. Let the position coordinate of a surface parallel to the bounding surfaces of the shell be s. Then, from Figure 1, the governing differential equation for heat transfer by conduction is

$$\frac{\rho(s)c(s)}{k(s)}\frac{\partial T(s,t)}{\partial t} = \frac{1}{k(s)A(s)}\frac{\partial(k(s)A(s))}{\partial s}\frac{\partial T(s,t)}{\partial s} + \frac{\partial^2 T(s,t)}{\partial s^2} \tag{6}$$

If the heat flux at the surfaces is specified as a function of surface temperature the boundary conditions may be specified as

$$q(T(s_1),t) = -k(s_1)\left(\frac{\partial T(s,t)}{\partial s}\right)_{s=s_1} \tag{7}$$

$$q(T(s_2),t) = -k(s_2)\left(\frac{\partial T(s,t)}{\partial s}\right)_{s=s_2} \tag{8}$$

where $q(T(s),t)$ is defined as being positive in the direction of increasing s.

To examine the stability of $T(s,t)$ as defined by eq 6, 7, and 8, assume a steady solution $T'(s)$ which satisfies eq 6, 7, and 8 and to $T'(s)$ add a small perturbation $\Delta T(s,t)$ such that

$$T(s,t) = T'(s) + \Delta T(s,t) \tag{9}$$

and such that

$$\Delta T(s,t) = e^{at}\mathrm{f}(s) \tag{10}$$

$T(s,t)$ will be stable for negative values of a and unstable for positive values of a. For a discussion of complex values of a see Wallis (1970). Determination of the stability of $T(s,t)$ can now be made by examining the possible solutions of eq 6, 7, and 8 of the form given by eq 9 and 10. If positive values of a are possible $T(s,t)$ will be unstable.

Substituting eq 9 and 10 into eq 6, 7, and 8 gives

$$\frac{\mathrm{d}\mathrm{f}(s)}{\mathrm{d}s} + \frac{1}{k(s)A(s)} \cdot \frac{\mathrm{d}[k(s)A(s)]}{\mathrm{d}s} \cdot \frac{\mathrm{d}\mathrm{f}(s)}{\mathrm{d}s} - \frac{\rho(s)c(s)}{k(s)} \cdot a \cdot \mathrm{f}(s) = 0 \tag{11}$$

$$\mathrm{f}(s_1)\frac{\partial q_1(T(s_1),t)}{\partial T(s_1)} = -k(s_1)\left(\frac{\mathrm{d}\mathrm{f}(s)}{\mathrm{d}s}\right)_{s=s_1} \tag{12}$$

and

$$\mathrm{f}(s_2)\frac{\partial q_2(T(s_2),t)}{\partial T(s_2)} = -k(s_2)\left(\frac{\mathrm{d}\mathrm{f}(s)}{\mathrm{d}s}\right)_{s=s_2} \tag{13}$$

Equation 11 is of the general form

$$\frac{\mathrm{d}^2\mathrm{f}(s)}{\mathrm{d}s^2} + \mathrm{g}(s)\frac{\mathrm{d}\mathrm{f}(s)}{\mathrm{d}s} - a \cdot \mathrm{p}(s) \cdot \mathrm{f}(s) = 0 \tag{14}$$

and a complete general solution is not known. However, if constant properties are assumed and if consideration is given only to surfaces such that

$$A(s) = Cs^n \tag{15}$$

eq 11 becomes

$$\frac{\mathrm{d}^2\mathrm{f}(s)}{\mathrm{d}s^2} + \frac{n}{s}\frac{\mathrm{d}\mathrm{f}(s)}{\mathrm{d}s} - \frac{a}{\alpha}\mathrm{f}(s) = 0 \tag{16}$$

or

$$\frac{\mathrm{d}}{\mathrm{d}s}\left[\left(s^n\frac{\mathrm{d}\mathrm{f}(s)}{\mathrm{d}s}\right)\right] - \frac{a}{\alpha}s^n\,\mathrm{f}(s) = 0 \tag{17}$$

Solutions to (17) are generally available in terms of exponential functions ($n = 0, a > 0$), trigonometric functions ($n = 0, a < 0$), Bessel functions ($n > 0, a < 0$), or modified Bessel functions ($n > 0, a > 0$). Two cases are of particular interest, the flat wall ($C = 1, n = 0$) and the circular cylinder ($C = 2\pi, n = 1$).

The Flat Wall

Consider a flat wall heated by convection on one surface and cooled by boiling on the other surface. Substituting x for s, eq 17 becomes

$$\frac{\mathrm{d}^2\mathrm{f}(x)}{\mathrm{d}x^2} - \frac{a}{\alpha}\mathrm{f}(x) = 0 \tag{18}$$

with boundary conditions

$$h\mathrm{f}(x_1) = k\left(\frac{\mathrm{d}\mathrm{f}(x)}{\mathrm{d}x}\right)_{x=x_1} \tag{19}$$

$$M\mathrm{f}(x_2) = -k\left(\frac{\mathrm{d}\mathrm{f}(x)}{\mathrm{d}x}\right)_{x=x_2} \tag{20}$$

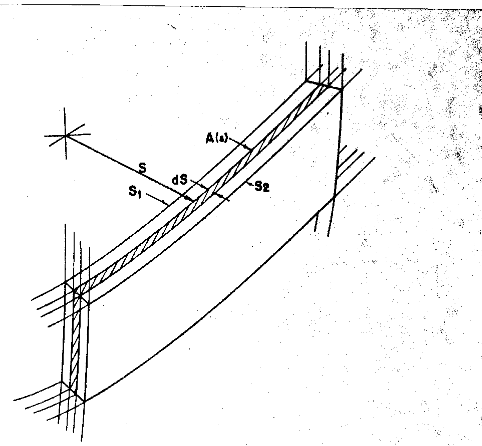

Figure 1. Math model

where h is the convective heat transfer coefficient at one surface and M is the slope of the boiling curve at a surface temperature $T(x_2)$. If eq 18 has solutions for positive values of a which satisfy boundary conditions (19) and (20), the wall will be thermally unstable; *i.e.*, the perturbation $\Delta T(x,t)$ will increase exponentially with time.

For positive values of a the general solution to (18) is

$$\mathrm{f}(x) = Ee^{\beta x} + Fe^{-\beta x} \tag{21}$$

where $\beta^2 = a/\alpha$. Substitution into boundary conditions (19) and (20) yields

$$e^{2\beta(x_2-x_1)} = \frac{\left(\beta - \frac{h}{k}\right)\left(\beta - \frac{M}{k}\right)}{\left(\beta + \frac{h}{k}\right)\left(\beta + \frac{M}{k}\right)} \tag{22}$$

If eq 22 has a real nonzero solution for β, a will be positive and instability will result.

To investigate eq 22, plot its left- and right-hand sides as functions of β. The right-hand side equals unity if $\beta = 0$, or $\pm\infty$, has zeros at $\beta = h/k$ and $\beta = M/k$ and poles at $\beta = -h/k$, $\beta = M/k$. Figure 2 shows that if h and M are both positive there is no solution other than $\beta = 0$.

If M is negative, however, the form of the solution depends on the relative magnitude of h and M. If $-M > h$ the result is shown in Figure 3. There are always two solutions and the situation is therefore unstable. This means that if

$$-M > h \tag{23}$$

boiling is unstable whatever the values of k and $(x_2 - x_1)$ may be. This is the same as the stability criterion given by eq 5 from the simplified analysis.

The interesting case occurs when $-M < h$. There is then the possibility of either stability or instability. Figure 4 shows the situation. Roots, other than the trivial case of $\beta = 0$, are possible only if the slope of the dashed line is less than the slope of $e^{2\beta(x_2-x_1)}$ at $\beta = 0$.

The slope of the dashed line may be evaluated by differentiating the right-hand side of eq 22. At $\beta = 0$ the value is $-2k(1/M + 1/h)$. The wall will therefore be stable if

$$2(x_2 - x_1) < -2k\left(\frac{1}{M} + \frac{1}{h}\right)$$

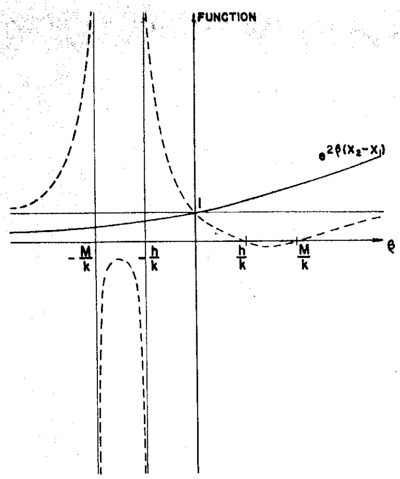

Figure 2. Solution of eq 22 for positive values of h and M

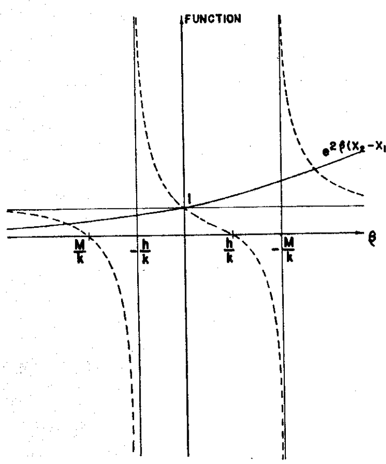

Figure 3. Solutions of eq 22 for positive h and negative M $(-M > h)$

That is, if

$$-M < \frac{1}{\dfrac{x_2 - x_1}{k} + \dfrac{1}{h}} \tag{24}$$

This is a "tighter" criterion for h than eq 23 and indicates that the wall resistance has a destabilizing effect. This criterion was used by Wallis (1965) in analyzing Berenson's data. In a later publication Audiutori (1965) included the wall resistance in a discussion of evaporator design. Neither author formalized the stability requirements imposed by eq 24.

Hollow Circular Cylinder

Replacing s by r for a hollow circular cylinder, eq 17 becomes

$$\frac{\mathrm{d}}{\mathrm{d}r}\left[r\frac{\mathrm{df}(r)}{\mathrm{d}r}\right] - \frac{a}{\alpha}\cdot r\cdot \mathrm{f}(r) = 0 \tag{25}$$

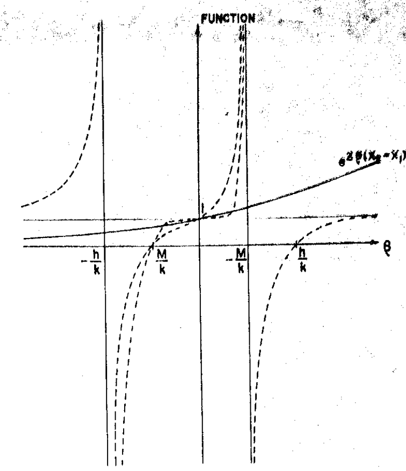

Figure 4. Solutions of eq 22 for positive h and negative M $(-M < h)$; two alternatives are shown

subject to boundary conditions

$$\mathrm{f}(r_1)\frac{\mathrm{d}q_1(R(r_1))}{\mathrm{d}(T(r_1))} = -kA(r_1)\left(\frac{\mathrm{df}(r)}{\mathrm{d}r}\right)_{r=r_1} \tag{26}$$

$$\mathrm{f}(r_2)\frac{\mathrm{d}q_2(R(r_2))}{\mathrm{d}(T(r_2))} = -kA(r_2)\left(\frac{\mathrm{df}(r)}{\mathrm{d}r}\right)_{r=r_2} \tag{27}$$

If the cylinder is heated on one surface by convection and cooled on the other by boiling, the boundary conditions become

$$h\mathrm{f}(r_1) = k\left(\frac{\mathrm{df}(r)}{\mathrm{d}r}\right)_{r=r_1} \tag{28}$$

$$M\mathrm{f}(r_2) = -k\left(\frac{\mathrm{df}(r)}{\mathrm{d}r}\right)_{r=r_2} \tag{29}$$

where h is the convective heat transfer coefficient and M is the slope of the boiling curve at a temperature $T(r_2)$. The solution to eq 25 for positive values of a is

$$\mathrm{f}(r) = AI_0(\beta r) + BK_0(\beta r) \tag{30}$$

where, again, $\beta^2 \equiv a/\alpha$. Application of this solution to boundary conditions (28) and (29) gives

$$\frac{hK_0(\beta r_1) + k\beta K_1(\beta r_1)}{hI_0(\beta r_1) - k\beta I_1(\beta r_1)} = \frac{MK_0(\beta r_2) - k\beta K_1(\beta r_2)}{MI_0(\beta r_2) + k\beta I_1(\beta r_2)} \tag{31}$$

or, in more convenient form

$$\frac{I_0(ZR) + b_2ZRI_1(ZR)}{I_0(Z) - b_1ZI_1(Z)} = \frac{K_0(ZR) - b_2ZRK_1(ZR)}{K_0(Z) + b_1ZK_1(Z)} \tag{32}$$

where $b_1 = k/hr_1$, $b_2 = k/Mr_2$, $Z = \beta r_1$, and $R = r_2/r_1$.

For positive values of b_1 and b_2 the left-hand side of eq 32 has a pole at $Z = I_0(Z)/(b_1\cdot I_1(Z))$ and no zeroes while the right-hand side has a single zero at $Z = K_0(ZR)/(b_2\cdot K_1(ZR))$ and no poles. The situation is shown in Figure 5. There is no solution for Z and the system will be stable.

For negative values of b_2 the left-hand side of eq 32 has a zero at $Z_0 = -I(ZR)/(b_2\cdot R\cdot I_1(ZR))$ and a pole at $Z_p = I_0(Z)/(b_1I_1(Z))$. The right-hand side has neither zeroes nor

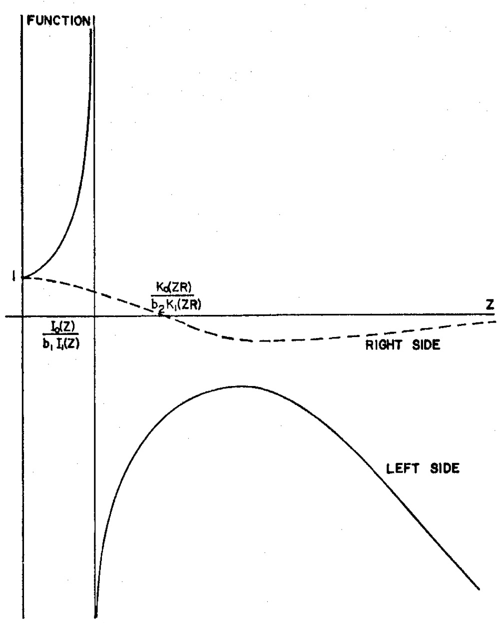

Figure 5. Solution of eq 32 for positive values of h and M

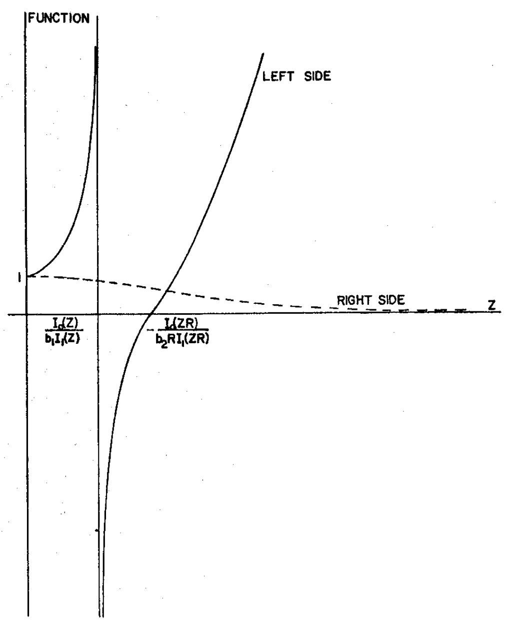

Figure 6. Solution of eq 32 for positive h and negative M ($Z_p < Z_0$)

poles, is always positive, and approaches zero as a limit with increasing Z. Two situations are possible, depending upon the relative magnitudes of Z_0 and Z_p. If $Z_p < Z_0$ the situation shown in Figure 6 occurs. There is always a real root and the system is unstable. If $Z_0 < Z_p$, on the other hand, Figure 7 shows that the system may be stable or unstable depending on

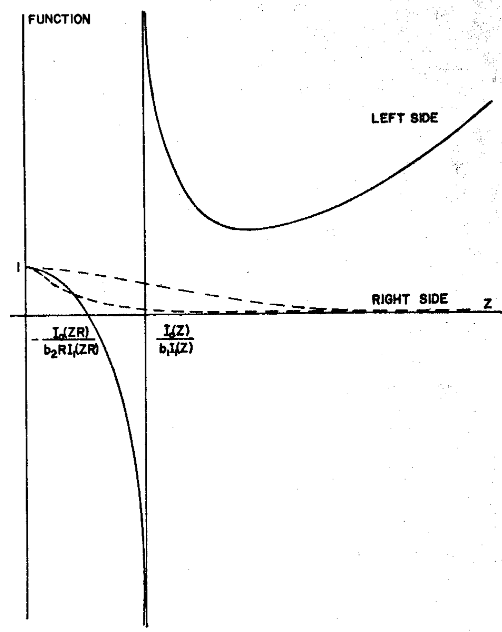

Figure 7. Solution of eq 32 for positive h and negative M ($Z_p > Z_0$); two alternatives are shown

the exact shape of the curves near $Z = 0$. In any case, a necessary (but not sufficient) condition for stability is

$$Z_0 < Z_p \qquad (33)$$

Numerical evaluation of Z_0 and Z_p for various values of b_1 and b_2 with $R > 1$ revealed that equality occurred in eq 33 for values of $-(b_1 + b_2)$ less than zero.

In order to establish a necessary condition for stability when eq 33 is satisfied, the behavior of both sides of eq 32 must be studied as $Z \to 0$. Both sides approach 1 with a first derivative of zero. Further differentiation reveals that the second derivatives are also equal. Rather than continue the lengthy differentiation process, it is simpler to examine the behavior of eq 32 near zero by substituting power series approximations for the Bessel functions. After considerable algebraic manipulation, retaining terms of second order only, eq 32 becomes

$$(b_1 + b_2 + \ln R) + \frac{Z^2}{2}\left[-\frac{1}{2}(R^2 - 1) + b_1\left\{-\ln R + \frac{1}{2}(R^2 - 1)\right\} + b_2\left\{R^2 \ln R - \frac{1}{2}(R^2 - 1)\right\} + b_1b_2/(R^2 - 1)\right] = 0 \quad (34)$$

Clearly, near $Z = 0$, $(b_1 + b_2 + \ln R) \to 0$.

Stability will occur if the solution of eq 34 yields imaginary values of Z, *i.e.*, if the coefficient of Z^2 has the same sign as $(b_1 + b_2 + \ln R)$. Accordingly, substitution of $b_2 = -(b_1 + \ln R)$ into the coefficient of $Z^2/2$ gives, for this coefficient

$$-b_1^2(R^2 - 1) + b_1(R^2 - 2R^2 \ln R - 1) + \frac{1}{2}(R^2 - 1)(\ln R - 1) - R^2 \ln^2 R$$

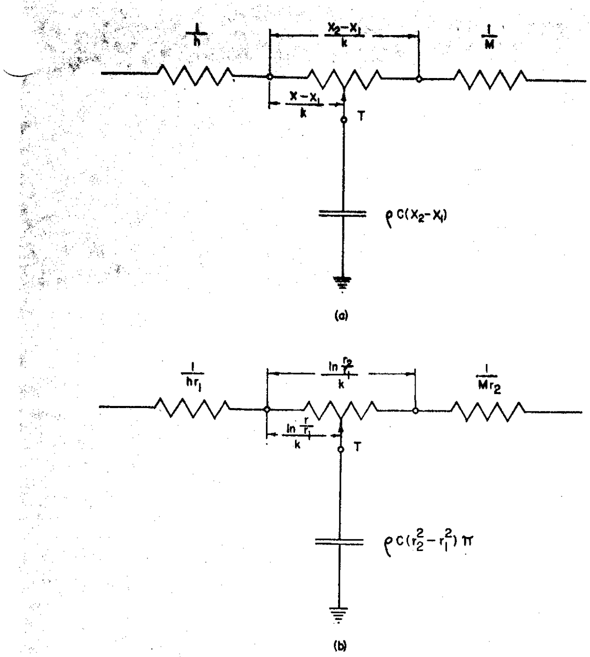

Figure 8. Schematic for lumped analysis: (a) flat wall; (b) circular cylinder

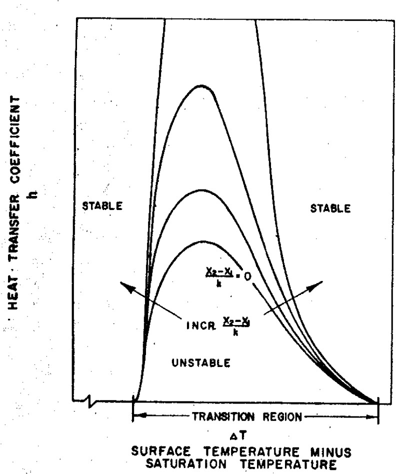

Figure 9. Stability envelopes for a convectively heated pool boiling system in the transition region

The maximum value of this coefficient will be <0 if

$$(R^2 - 2R^2 \ln R - 1)^2 < -4(R^2 - 1) \times \left[\frac{1}{2}(R^2 - 1)(\ln R - 1) - R^2 \ln^2 R\right]$$

i.e., if

$$-(R^2 - 1)^2 + 2 \ln R(1 + 2R^2 \ln R - R^4) < 0$$

The first term in the above expression is always negative and the second term is negative for $R > 1$. Therefore, the coefficient of Z^2 in eq 34 is negative near the stability boundary for all values (positive or negative) of b_1 and

$$\text{for stability: } b_1 + b_2 + \ln R < 0 \qquad (35a)$$

$$\text{for instability: } b_1 + b_2 + \ln R > 0 \qquad (35b)$$

Remembering that b_2 is negative, eq 35a may be written as

$$-\frac{k}{Mr_2} > \frac{k}{hr_1} + \ln\frac{r_2}{r_1} \qquad (36)$$

Equation 36 is a "tighter" criterion for stability than eq 33 and is, apparently, both necessary and sufficient.

The arguments leading to eq 35 are equally valid if b_1, rather than b_2, is negative. In this case the stability condition for boiling inside the cylinder may be obtained by interchanging the roles of M and h and is

$$-\frac{k}{Mr_1} > \frac{k}{hr_2} + \ln\frac{r_2}{r_1} \qquad (37)$$

Alternate Analysis, Lumped Parameter

An alternate approach is to approximate the system by lumping the heat capacity of the wall at a particular place. For a flat wall the lumped capacitance can be placed at any point in the wall as shown in Figure 8a. Wallis (1970) has shown that the resulting analysis indicates instability in the range

$$\left(\frac{1}{h} + \frac{x_2 - x_1}{k}\right) > -\frac{1}{M} > \left(\frac{x_2 - x_1}{k} - \frac{x - x_1}{k}\right) \qquad (38)$$

The lumped model agrees with the continuum model only if the capacitance is lumped at x_2 so that $x = x_2$. Otherwise, the lumped model produces the spurious conclusion indicated by the right-hand inequality in eq 38. Applying the same type of analysis to a circular cylinder, with reference to Figure 8b, again gives agreement with the continuum analysis only if the capacitance is lumped at the boiling surface.

Experimental Verification

Obviously, eq 24, 36, and 37 place severe limitations on the use of convectively heated surfaces in transition region boiling. Values of M near the peak heat flux seem to be typically about -1×10^4 Btu/hr ft²°F. Referring to eq 24, $(x_2 - x_1)/k$, the wall resistance must, then, be less than about 1×10^{-4} hr ft²°F/Btu for stability throughout the transition region even if $h \rightarrow \infty$. A flat steel plate 0.1 in. thick has a wall resistance of about 5×10^{-4} hr ft²°F/Btu and a flat copper plate 0.25 in. thick has a wall resistance of about 1×10^{-4} hr ft²°F/Btu. A stability envelope based on the application of eq 24 to a typical pool boiling curve will be similar to Figure 9 with $(x_2 - x_1)/k > 0$. Systems of this type using thick surfaces are obviously inappropriate for investigations of the transition region. This effect is evident in the data presented by Berenson (1962) and in the data shown in Figure 10.

Equation 24 was applied to Berenson's data for runs 17 and 22 and for runs 2 and 3 for pentane boiling on the top surface of a copper disk. The disk was 2.25 in. thick and was heated by steam condensing on the bottom surface. Assuming the condensation heat transfer coefficient to be infinite, and using Berenson's straight line interpretation of the transition region on a log–log plot, the unstable range for runs 17 and 22 was predicted to be 16–37°F. No data were reported in the range of 16F–43°F. The results were similar for runs 2 and 3 with a predicted range of 85–107°F and a measured range of 85–

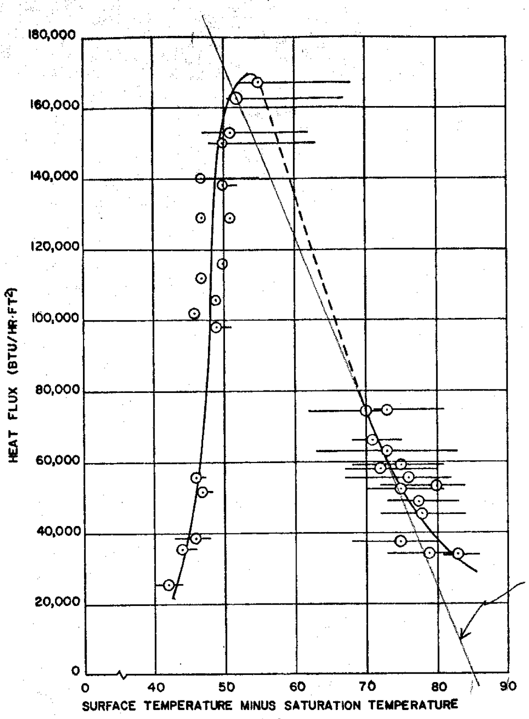

Figure 10. Pool boiling curve for methanol on a copper surface

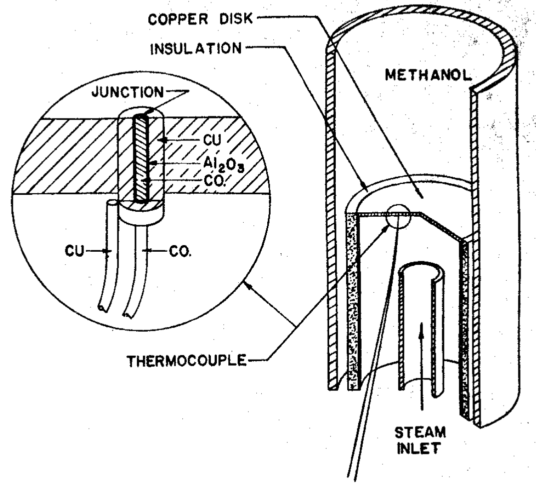

Figure 11. Schematic diagram of system used to boil methanol

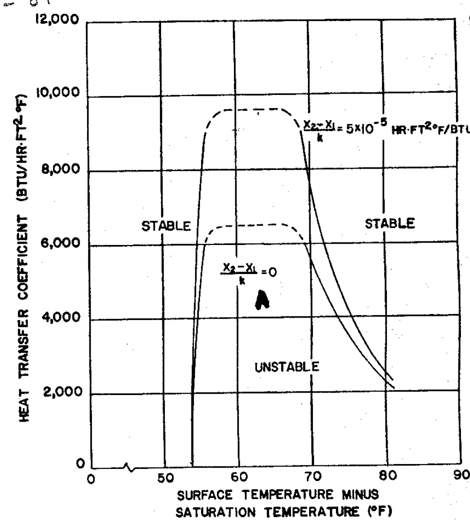

Figure 12. Stability envelope for pool boiling of methanol on a copper disk heated by convection

105°F. The limitations imposed on Berenson's data by the high wall resistance were first discussed by Wallis in his comments on Audiutori's original paper.

The data shown in Figure 10 were obtained from the apparatus shown schematically in Figure 11. The heated surface was a copper disk 3 in. in diameter and 0.125 in. thick. The disk was heated by saturated steam impinging on the bottom surface and was cooled by boiling methanol on the top surface. The top surface temperature was measured by using a small coaxial copper–constantan thermocouple as shown in the schematic diagram. Heat fluxes were determined by measuring the flow rate and temperature rise of cooling water flowing through a reflux condenser attached directly to the top of the boiling chamber. The significant aspect of the data obtained from this test is the complete absence of data between 55 and 70°F superheat. Figure 12 shows the stability envelopes obtained by applying eq 24 to the boiling curve shown in Figure 10, assuming $(x_2 - x_1)/k = 0.0$ and also taking $(x_2 - x_1)/k = 5.0 \times 10^{-5}$ hr ft²°F/Btu, the approximate resistance of the copper disk. The shape of the stability envelope depends strongly on the exact shape of the boiling curve and, considering the scatter associated with most boiling measurements, there may be many different curves drawn from the same data. Figure 12 does, however, clearly demonstrate the significance of the thermal resistance of the surface in the determination of the thermal stability of the design.

Conclusion

The thermal stability of a flat wall and a cylindrical wall, heated by convection and cooled by boiling, has been investigated. It has been shown that certain relative magnitudes of system parameters may result in instability when transition boiling occurs, *i.e.*, when the heat flux and temperature are such that the slope of the boiling curve is negative. Specific conclusions are as follows.

1. For thermal stability of a flat wall

$$-M < \frac{1}{\frac{x_2 - x_1}{k} + \frac{1}{h}} \qquad (24)$$

2. For thermal stability of a cylindrical wall with boiling on the outside

$$-\frac{k}{Mr_2} > \frac{k}{hr_1} + \ln\frac{r_2}{r_1} \qquad (36)$$

3. For thermal stability of a cylindrical wall with boiling on the inside

$$-\frac{k}{Mr_1} > \frac{k}{hr_2} + \ln\frac{r_2}{r_1} \qquad (37)$$

4. The above conclusions may be reached by a lumped parameter analysis only if the thermal capacitance of the wall is taken to be concentrated at the boiling surface.

Besides obtaining solutions for these simple practical cases this paper has shown how problems of this type may be approached by two techniques: lumped-parameter system analysis and eigenfunction method using distributed parameters. These results have been verified using Liapunov methods by Pritchard (1971) and should help to establish a thorough and versatile mathematical methodology for dealing with the practically important, but neglected, subject of thermal stability. It is an obvious step to extend the methods to deal with systems (such as nuclear reactor fuel elements) with variable properties, distributed heat sources, and composite walls.

Acknowledgment

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Nomenclature

A = area
a = a constant
b = a constant
b_1 = dimensionless parameter, k/hr_1
b_2 = dimensionless parameter, k/Mr_2
C = a constant
c = specific heat, also used as a constant
E = a constant
F = a constant
f = a function
g = a function
h = convective heat transfer coefficient, *i.e.*, rate of change of heat flux with temperature difference, assumed constant
I_0 = modified Bessel function of the first kind of order zero
I_1 = modified Bessel function of the second kind of order one
K_0 = modified Bessel function of the second kind of order zero
K_1 = modified Bessel function of the second kind of order one
k = thermal conductivity
M = rate of change of heat flux with temperature difference for a boiling process; a function of surface temperature
n = a constant
p = spatial function
q = heat flux
R = dimensionless parameter, r_2/r_1
r = spatial coordinate, cylindrical coordinate system
s = spatial coordinate, in direction of heat transfer
T = temperature
ΔT = temperature perturbation
t = time
x = spatial coordinate, rectangular coordinate system
Z = dimensionless parameter βr_2

Greek Letters

α = thermal diffusivity
β = defined as $\sqrt{a/\alpha}$
ρ = density

Subscripts

1, 2 = the two boundaries of the wall
0, p = zeroes and poles
f = fluid
w = wall
o, i = output and input

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