Why laws such as Young's law and Ohm's law should be abandoned, and the laws that should replace them.

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Abstract

The history of engineering methodology is reviewed, and modern methodology is critically examined. The examination reveals that the manner in which the dimensional homogeneity of parametric equations is currently achieved should be abandoned because it:

- Is based on an *irrational* premise.
- Requires *contrived* parameters such as elastic modulus (the ratio of stress to strain) and electrical resistance (the ratio of electromotive force to electric current).
- Makes it *impossible* to have laws that are *always* obeyed.
- Results in laws that are obeyed if the behavior of concern is proportional, but are *not* obeyed if the behavior of concern is nonlinear.
- Requires *two* methodologies in each branch of engineering:
    - One methodology for problems that concern proportional behavior, and are solved using laws such as Young's law and Ohm's law.
    - A second methodology for problems that concern nonlinear behavior, and are solved *without* using laws such as Young's law and Ohm's law.

In this monograph, dimensional homogeneity is achieved in a manner that results in laws that are *always* obeyed, and *one* methodology that applies to *both* proportional and nonlinear behavior.

The laws are presented, and their application is demonstrated in example problems that concern proportional and nonlinear behavior in stress/strain engineering, electrical engineering, and heat transfer engineering.

The problems reveal the marked improvement in methodological scope and simplicity that results when laws such as Young's law and Ohm's law are replaced by laws that are always obeyed.

1. Introduction

It is axiomatic that parametric equations must be dimensionally homogenous. Although the manner in which homogeneity is now achieved may seem intuitive, it is open to question, as are all things in science.

The manner in which dimensional homogeneity is achieved is extremely important because it has a *profound* influence on engineering methodology.

For 2000 years, scientists such as Euclid, Galileo, and Newton agreed that different dimensions *cannot* rationally be multiplied or divided.

Dimensional homogeneity was achieved by requiring that parametric equations consist of ratios in which the numerator and denominator are the *same* parameter. Thus equations are homogeneous because *each ratio* is dimensionless.

Because it was agreed that different dimensions cannot be multiplied or divided, parametric equations such as "force equals mass times acceleration" were considered irrational.

Two centuries ago, Fourier performed experiments to determine the laws of convective and conductive heat transfer. (In Fourier's day, a law was a parametric equation that is *always* obeyed. Today, a law is a parametric equation that may or may *not* be obeyed, such as Young's law and Ohm's law.)

Fourier recognized that the heat transfer behavior he induced could be described by quantitative, homogeneous equations *only* if dimensions could be multiplied and divided.

Fourier convinced his contemporaries that, for 2000 years, the scientific community was *wrong* in maintaining that different dimensions cannot be multiplied or divided. He convinced his contemporaries that it is both *rational and necessary* to multiply and divide different dimensions. Fourier (1822a) is generally credited with the modern view of homogeneity.

2. Nomenclature

- Parameter symbols represent numerical value *and* dimension if they refer to modern engineering methodology.
- Parameter symbols represent numerical value but *not* dimension if they refer to modified engineering methodology. Where appropriate, dimension units that underlie parameter symbols are specified in nomenclatures within the text.

2.1 Symbols

a acceleration

b a constant

c a constant

E elastic modulus σ/ε, or electromotive force

f force

f{ } denotes function of

h heat transfer coefficient $q/\Delta T_{BL}$

I electric current

k thermal conductivity $q/(\mathrm{d}T/\mathrm{d}x)$

L length of a standard copper wire

m mass

q heat flux

R electrical resistance E/I

t thickness

T temperature

U overall heat transfer coefficient $q/\Delta T_{total}$

x distance

ε strain

σ stress

2.2 Subscripts

BL refers to boundary layer

cond refers to conductive

conv refers to convective

parallel refers to type of electrical connection

wall refers to heat transfer wall

3. The history of dimensional homogeneity.

For 2000 years, scientists such as Euclid, Galileo, and Newton agreed that different dimensions *cannot* rationally be multiplied or divided. The 2000 year view of homogeneity is reflected in the following:

Algebraically, speed is now represented by a "ratio" of space traveled to time elapsed. For Euclid and Galileo, however, no true ratio could exist at all except between two magnitudes of the same kind. Drake (1974)

3.1 Dimensional homogeneity in Galileo's Theorem VI, Proposition VI.

The following equation is Galileo's Theorem VI, Proposition VI:

If two particles are carried at a uniform rate, the ratio of their speeds will be the product of the ratio of the distances traversed by the inverse ratio of the time-intervals occupied. Galileo (1638)

Galileo's equation reflects the 2000 year view of homogeneity. It includes the ratios speed to speed, distance traversed to distance traversed, and time-interval to time-interval. The ratios are dimensionless and therefore can be multiplied and divided, and the equation is dimensionless and homogeneous.

Galileo saw no conflict between the concept of speed, and the view that distance cannot be divided by time. To Galileo, distance and time were necessary to quantify speed, but speed had nothing to do with *dividing* distance by time.

In Galileo's view, and in the view of the science community for 2000 years, dividing distance by time is irrational.

3.2 Dimensional homogeneity in Newton's version of the second law of motion.

Equation (1) is generally referred to as Newton's second law of motion. Newton's famous treatise *The Principia,* is cited.

$$f = ma \tag{1}$$

However, Eq. (1) is *not* the second law of motion conceived by Newton and published in *The Principia*. Newton's version of the second law is described in the following from *The Principia*:

A change in motion is proportional to the motive force impressed and takes place along the straight line in which that force is impressed. Newton (1726)

Note that Newton's version is *not* an equation, it is *not* quantitative, and it does *not* include mass. It is a proportion, and it conforms to the 2000 year view of homogeneity in that it does not require that mathematical operations be performed on dimensions.

Newton describes the application of his version of the second law in the following:

If some force generates any motion, twice the force will generate twice the motion, and three times the force will generate three times the motion.
Newton (1726)

(Force 1/Force 2) = (Motion 1/Motion 2) (2)

In application, Newton's version reflects the 2000 year view of dimensional homogeneity because both ratios in Eq. (2) contain the same parameter in the numerator and the denominator.

3.3 Dimensional homogeneity in Ohm's version of Ohm's law.

Equation (3) is generally referred to as Ohm's law. Ohm's famous treatise, *The Galvanic Circuit Investigated Mathematically* is cited.

$$E = IR \qquad (3)$$

However, Eq. (3) is *not* the law conceived by Ohm and published in *The Galvanic Circuit Investigated Mathematically*. Ohm's version of Ohm's law is described in the following:

. . . the current in a galvanic circuit is directly as the sum of all the tensions, and inversely as the entire reduced length of the circuit . . .

$$I = E/L \qquad (4)$$

Ohm (1827)

L is the "reduced length of the circuit"—ie L is the length of a copper wire of a standard diameter that exhibits the same electrical behavior as the circuit.

Equation (4) is *not* homogeneous. Presumably, Ohm saw no reason to comply with Fourier's view that scientific parametric equations must be homogeneous, or with the 2000 year view that equations must consist of ratios in which the same parameter is in the numerator and the denominator.

Ohm's inhomogeneous Eq. (4) was used for decades, and with excellent practical result, as indicated in the following published three decades after the publication of Ohm (1827):

(Reduced length) is a very convenient mode of expressing the resistance to conductability, presented by the whole or by a part of a circuit, to reduce it to that which would be presented by a certain length of wire of a given nature and diameter.
De La Rive (1856).

3.4 Why Clerk Maxwell (1831-1879) would abandon Ohm's law and electrical resistance.

In the following, Clerk Maxwell explains why Eq. (3) is a law, and why electrical resistance has scientific value.

. . . the resistance of a conductor . . . is defined to be the ratio of the electromotive force to the strength of the current which it produces. The introduction of ***this term would have been of no scientific value*** *unless Ohm had shown, as he did experimentally, that . . . it has a definite value which is altered only when the nature of the conductor is altered.*

In the first place, then, the resistance of the conductor is ***independent*** *of the strength of the current flowing through it.*

The resistance of a conductor may be measured to within one ten thousandth, . . . and so many conductors have been tested that our assurance of the truth of Ohm's Law is now very high.
Maxwell (1873)

In summary:

- Electrical resistance R is defined to be E/I. Therefore R and E/I are *identical* and *interchangeable*.
- Many conductors were tested with great accuracy, and *all* conductors tested indicated that E/I is independent of I—ie indicated that *all* conductors tested obeyed Ohm's law.
- R has scientific value because it is independent of I for *all* conductors.
- Eq. (3) is a true law because it is *always* obeyed.

Maxwell's words indicate that if some conductors did *not* obey Ohm's law, then Ohm's law would *not* be a true law, and electrical resistance E/I would have *no scientific value*.

If Maxwell were alive, he would surely abandon Ohm's law and electrical resistance because of the many conductors that do *not* obey Ohm's law.

4. Fourier's heat transfer laws, and his revolutionary view of homogeneity.

Fourier's famous treatise, *The Analytical Theory of Heat* (1822), presents many of Fourier's contributions to engineering science in general, and to heat transfer science in particular. Fourier described the purpose of his heat transfer research in the following:

Primary causes are unknown to us; but are subject to simple and constant laws, which may be discovered by observation . . .

The object of our work is to set forth the mathematical laws which this element (ie heat transfer) *obeys.* Fourier (1822b)

4.1 Fourier's experiments and results.

Fourier performed comprehensive experiments in both convective and conductive heat transfer. From data he had obtained, Fourier induced that:

- Convective heat flux q_{conv} is generally proportional to boundary layer temperature difference ΔT_{BL}.

$$q_{conv} \; \alpha \, \Delta T_{BL} \tag{5}$$

- Conductive heat flux q_{cond} is generally proportional to temperature gradient dT/dx.

$$q_{cond} \; \alpha \; \mathrm{d}T/\mathrm{d}x \tag{6}$$

Presumably, Galileo and Newton would have been satisfied with Proportions (5) and (6). But Fourier was not satisfied with proportions—he wanted quantitative, homogeneous *equations* that *always* applied, and would be considered *laws*.

The transformation of Proportions (5) and (6) results in Eqs. (7) and (8) in which b and c are constants.

$$q_{conv} = b\,\Delta T_{BL} \tag{7}$$

$$q_{cond} = c(\mathrm{d}T/\mathrm{d}x) \tag{8}$$

Fourier was not satisfied with Eqs. (7) and (8) because they are *not* dimensionally homogeneous.

4.2 Fourier's revolutionary view of dimensional homogeneity, and why he could not prove that his view is scientifically correct.

Fourier described his revolutionary view of dimensional homogeneity in the following:

. .every undetermined magnitude or constant has one dimension proper to itself, and the terms of one and the same equation could not be compared, if they had not the same exponent of dimensions. Fourier (1822a)

In other words, parameters and constants in equations have dimension, and the terms in an equation cannot be compared unless all terms have the same dimension—ie unless the equation is dimensionally homogeneous.

The dimension of a term is determined by multiplying and dividing the dimensions in the term, as described in Fourier (1822c). Fourier continued:

(This view of homogeneity) *is the equivalent of the fundamental lemmas which the Greeks have left us without proof.* Fourier (1822a)

Fourier claimed that his view of homogeneity was scientifically correct because *"it is the equivalent of the fundamental lemmas* (axioms) *which the Greeks have left us".*

He could not prove that his view was scientifically correct because "*the Greeks have left us without proof*", and presumably Fourier could not prove that the lemmas left by the Greeks were correct.

4.3 How Fourier transformed Eqs. (7) and (8) from inhomogeneous to homogeneous.

Equations (7) and (8) are inhomogeneous. To make them homogeneous, Fourier assigned the required dimensions to constants b and c, and replaced b and c with h and k, resulting in Eqs. (9) and (10).

$$q_{conv} = h\Delta T_{BL} \tag{9}$$

$$q_{cond} = k(\mathrm{d}T/\mathrm{d}x) \tag{10}$$

(Note that Fourier homogenized Eqs. (7) and (8) by the *unwarranted* transformation of *dimensionless* constants b and c into *dimensioned* parameters h and k.)

Fourier and his contemporaries accepted Eqs. (9) and (10) as *laws*—ie as equations that state "q_{conv} is *always* proportional to ΔT; h is *always* a constant of proportionality; q_{cond} is *always* proportional to dT/dx; k is *always* a constant of proportionality".

4.4 Fourier's valid claim of priority.
Fourier validly claimed priority for the concepts of heat transfer coefficient and thermal conductivity, and for Eqs. (9) and (10), in the following:

I have induced these laws (ie Eqs. (9) and (10)) *from prolonged study and attentive comparison of the facts known up to this time: all these facts I have observed afresh in the course of several years with the most exact instruments that have hitherto been used.*
Fourier (1822d)

(American heat transfer texts inappropriately refer to Eq. (9) as "Newton's law of cooling".)

4.5 Why Fourier's contemporaries accepted his revolutionary view of homogeneity.
Even though Fourier could not prove that his revolutionary view of homogeneity was scientifically correct, his contemporaries accepted his view because his treatise demonstrates a *quantitative* understanding of heat transfer, whereas his contemporaries had *no* understanding of heat transfer.

For example, Biot (1804) includes the following conclusion about conductive heat transfer:

Thus it is physically impossible to heat to one degree the end of an iron bar of two metres or six feet in length by heating the other end, because it would melt before this.

Fourier explained why Biot's conclusion was not correct:

. . . this result depends on the thickness of the (bar) employed. If it had been greater, the heat would have been propagated to a greater distance . . . We can always raise by one degree the temperature of one end of a bar of iron, by heating the solid at the other end; we need only give the radius of the base a sufficient length; which is, we may say, evident, and of which besides a proof will be found in the (quantitative) solution of the problem (given below).
Fourier (1822e)

Fourier's knowledge of heat transfer was so complete that he was able to calculate the temperature distribution throughout the bar, whereas his contemporaries did not even know which parameters influence conductive heat transfer.

Fourier explained the critical importance of dimensional homogeneity:

If we did not make a complete analysis of the elements of the problem, we should obtain an equation not homogeneous, and, a forteriori, we should not be able to form the equations which express the movement of heat in more complex cases.
Fourier (1822e)

5. The manner in which dimensional homogeneity is currently achieved, and why laws such as Young's law and Ohm's law are *in*homogeneous.
The manner in which dimensional homogeneity is currently achieved is described in the following:

- Parameter symbols in equations represent numerical value and dimension.
- Dimensions may be multiplied and divided. They may *not* be added or subtracted.
- Dimensions must *not* be assigned to constants. As explained by Langhaar (1951), *Dimensions must not be assigned to numbers, for then any equation could be regarded as dimensionally homogeneous.*
- The dimension of a term in an equation is determined by multiplying and dividing the dimensions of the parameters in the term.

Fourier's view of homogeneity differs from the modern view in one respect: Fourier's view *allows* dimensions to be assigned to constants, whereas the modern view does *not* allow it. In Fourier's words:

. . .every undetermined magnitude ***or constant*** *has one dimension proper to itself.* Fourier (1822b)

As noted in Section 4.3, the homogeneity of Fourier's laws of heat transfer *requires* that dimensions be assigned to *constants b and c* in Eqs. (7) and (8). Similarly, the homogeneity of laws such as Young's law and Ohm's law *requires* that dimensions be assigned to *constants*.

Laws such as Young's law and Ohm's law and Fourier's laws of heat transfer are *in*homogeneous because their homogeneity is achieved by assigning dimensions to *constants,* in violation of the modern view of homogeneity.

6. Why the modern view of homogeneity is irrational.

The modern view of homogeneity is irrational because it is based on the irrational premise that dimensions *can* be multiplied and divided, but *cannot* be added or subtracted.

Multiplication is repeated addition. "Multiply six times eight" *means* "Add six eights." Because multiplication is repeated addition, things that cannot be added cannot be multiplied. It is irrational for the modern view of homogeneity to maintain that dimensions *cannot* be added, but *can* be multiplied.

Also, "Multiply six times eight" *means* "add six eights". Therefore "multiply meters times kilograms" *means* "add meter kilograms". Because "add meter kilograms" has no meaning, it is irrational to multiply dimensions.

"Divide twelve by four" *means* "how many fours are in twelve?" Therefore "divide meters by minutes" means "how many minutes are in meters?" Because "how many minutes are in meters?" has no meaning, it is irrational to divide dimensions.

In modern engineering, the multiplication and division of dimensions are indicated symbolically. But that does not prove that dimensions can actually be multiplied and divided.

7. Why laws such as Young's law and Ohm's law are irrational.

It is self-evident that dissimilar things *cannot* be proportional.

- Because alligator and house are dissimilar, they *cannot* be proportional. Therefore it is irrational to state that alligator is proportional to house.
- Because stress and strain are dissimilar, they *cannot* be proportional. Therefore it is irrational to state that stress is proportional to strain.
- Young's law is *irrational* because it states that stress is proportional to strain, and similarly for Ohm's law, and Fourier's laws of heat transfer.

"Stress is proportional to strain" is understood to mean "The *numerical value* of stress is proportional to the *numerical value* of strain". The *understood* meaning of the statement is rational, but the statement itself is irrational. And similarly for laws such as Young's law, Ohm's law, Fourier's laws of heat transfer, and other proportional laws.

8. Fourier's error of induction.

There is an error in Fourier's induction. From data he had obtained, Fourier induced

> Convective heat flux is proportional to boundary layer temperature difference.

But convective heat flux *cannot* be proportional to boundary layer temperature difference because they are dissimilar.

The proportional relationship that Fourier observed in the data was *not* between heat flux and temperature difference. It was between *numerical value* of heat flux and *numerical value* of temperature difference. Therefore Fourier *should* have induced

> The *numerical value* of convective heat flux is proportional to the *numerical value* of boundary layer temperature difference.

9. How Fourier's error of induction affected his methodology and conclusions.

If Fourier had correctly induced that the *numerical value* of convective heat flux is proportional to the *numerical value* of boundary layer temperature difference, his methodology and conclusions would have been affected in the following ways:

- His parameter symbols would have represented numerical value but *not* dimension.
- He would have stated that the dimension units that underlie parameter symbols in quantitative equations must be specified in accompanying nomenclatures.
- He would *not* have assigned dimensions to constants *b* and *c* in Eqs. (7) and (8) because all symbols in both equations would have represented numerical value but *not* dimension. Therefore Eqs. (7) and (8) would have been dimensionally homogeneous *as written*.
- He would *not* have conceived the modern view of dimensional homogeneity because it would not have been required in order to make Eqs. (7) and (8) homogeneous.
- He would *not* have conceived "heat transfer coefficient" or "thermal conductivity" because they

would not have been required in order to make Eqs. (7) and (8) homogeneous.

- He *would* have accepted Eq. (7) as the law of convective heat transfer, and Eq. (8) as the law of conductive heat transfer.
- His view of homogeneity would have been that parametric equations are *inherently* homogeneous because parameter symbols represent numerical value but *not* dimension.

10. The parameter symbolism that should replace the modern parameter symbolism.
Parameter symbols in equations represent numerical value, but *not* dimension. Dimension units that underlie parameter symbols in quantitative equations must be specified in accompanying nomenclatures.

11. The view of homogeneity that should replace the modern view.
Parametric equations are *inherently* homogeneous because they contain *only* numbers.

12. The laws that should replace laws such as Young's law and Ohm's law.
The laws that should replace laws such as Young's law and Ohm's law are analogs of Eq. (11).

$$y = \mathrm{f}\{x\} \qquad (11)$$

Equation (11) states that the numerical value of *y* is an unspecified function of the numerical value of *x*, and the unspecified function includes *all* forms of behavior—proportional, linear, and nonlinear.

Equation (12) should replace Young's law. Eq. (12) states that the numerical value of stress is an unspecified function of the numerical value of strain, and the unspecified function includes *all* forms of behavior.

$$\sigma = f\{\varepsilon\} \qquad (12)$$

Equation (12) is appropriately named "the *law* of material behavior" because the numerical value of stress is *always* a function of the numerical value of strain, and therefore Eq. (12) is *always* obeyed. Equation (12) is used to solve problems that concern elastic behavior *and* inelastic behavior.

It is important to note that, if the symbols in Eq. (12) represent numerical value *and* dimension, Eq. (12) is *in*homogeneous and therefore unacceptable.

Equations such as Eq. (12) are acceptable *only* if parameter symbols represent numerical value but *not* dimension.

Eq. (13) should replace Ohm's law. Equation (13) states that the numerical value of electromotive force is an unspecified function of the numerical value of electric current, and the unspecified function includes *all* forms of behavior.

$$E = \mathrm{f}\{I\} \qquad (13)$$

Eq. (13) is appropriately named "the *law* of resistive electrical behavior" because the numerical value of *E* is *always* a function of the numerical value of *I*. Therefore Eq. (13) is *always* obeyed.

Equation (13) is used to solve problems that concern resistive devices that obey Ohm's law, *and* resistive devices that do *not* obey Ohm's law.

13. The parameters that should replace contrived parameters such as elastic modulus and electrical resistance.
No parameters should replace parameters such as *E* (the ratio of stress to strain in the elastic region) and *R* (the ratio of electromotive force to electric current) because they are irrelevant if parameter symbols represent numerical value but *not* dimension.

The *only* reason modern engineering requires contrived parameters such as *E* and *R* is so that proportional laws can be expressed homogeneously using parameter symbols that represent numerical value *and* dimension.

If parameter symbols represent numerical value but *not* dimension, parametric equations are *inherently* homogeneous, and contrived parameters such as *E* and *R* are irrelevant.

If Young's law is used to solve problems, solutions involve parameters σ, ε, and *E*. If Eq. (12) is used to solve problems, solutions involve parameters σ and ε. Since problems are readily solved *without* using parameter *E*, there is no reason to replace it.

If Ohm's law is used to solve problems, solutions involve *E*, *I*, and *R*. If Eq. (13) is used to solve problems, solutions involve *E* and *I*. Since problems are readily solved *without* using *R*, there is no reason to replace it.

14. The "dimensional equations" widely used in much of the twentieth century.
When modern engineering science is modified to correct for the effect of Fourier's error, parameter symbols in equations represent numerical value but *not* dimension. The dimensions that underlie parameter symbols in quantitative equations are specified in accompanying nomenclatures.

Although such equations may now seem bizarre, they were widely used in much of the twentieth century. They were generally referred to as "dimensional equations", as exemplified in the following:

*For the turbulent flow of gases in straight tubes, the following **dimensional equation** for forced convection is recommended for general use:*

$$h = 16.6\ c_p(G')^{0.8}/D_i')^{0.2} \quad (14)$$

where c_p is the specific heat of the gas at constant pressure, B.T.u./(lb.)(°F), G′ is the mass velocity, expressed as lb. of gas/sec./sq. ft, . . . Perry (1950)

Eq. (14) would obviously be inhomogeneous if parameter symbols represented numerical value *and* dimension. Note that the dimensions that underlie the symbols are specified in the accompanying nomenclature, as required if parameter symbols represent numerical value but *not* dimension.

Equations in which parameter symbols represent numerical value but *not* dimension are still used, but not widely. For example, Holman (1997) lists two tables of "simplified equations" such as Eq. (15) from Table (7-2) and Eq. (16) from Table (9-3).

$$h = 1.42(\Delta T/L)^{1/4} \quad (15)$$

$$h = 5.56(\Delta T)^3 \quad (16)$$

Tables (7-2) and (9-3) note that the dimension units that underlie the parameter symbols are watts, meters, and degrees Centigrade.

15. The methodology that results when modern engineering methodology is modified to correct for the impact of Fourier's error of induction.
When modern engineering methodology is modified to correct for the impact of Fourier's error of induction, the resultant methodology is described by the following:

- Parameter symbols in equations represent numerical value but *not* dimension. Dimension units that underlie parameter symbols in quantitative equations are specified in accompanying nomenclatures.
- Parametric equations are *inherently* homogeneous because they contain only numbers.
- There are *no* proportional laws such as Young's law and Ohm's law and Fourier's laws of convective and conductive heat transfer.
- Laws such as Young's law and Ohm's law and Fourier's laws of heat transfer are replaced by laws that are *always* obeyed, such as the law of material behavior, Eq. (17), the law of resistive electrical behavior, Eq. (18), and the law of convective heat transfer behavior, Eq. (19).

$$\sigma = f\{\varepsilon\} \quad (17)$$

$$E = f\{I\} \quad (18)$$

$$q_{conv} = f\{\Delta T_{BL}\} \quad (19)$$

- There are *no* contrived parameters such as
  - Elastic modulus, the ratio of stress to strain.
  - Electrical resistance, the ratio of electromotive force to electric current.
  - Heat transfer coefficient, the ratio of heat flux to temperature difference.
- There are *no* equations such as Eqs. (20) and (21).

$$R_{parallel} = 1/(1/R_2 + 1/R_3 + 1/R_4) \quad (20)$$

$$U = 1/(1/h_1 + t_{wall}/k_{wall} + 1/h_2) \quad (21)$$

- Equations such as Eqs. (20) and (21) have been replaced by equations such as Eqs. (22) and (23).

$$I_{parallel} = I_2 + I_3 + I_4 \quad (22)$$

$$\Delta T_{total} = \Delta T_1 + \Delta T_{wall} + \Delta T_2 \quad (23)$$

- Equations (20) and (22) are *identical*. Equation (20) is used *only* if the relationship between *E* and *I* is proportional. Equation (22) is used whether the relationship between *E* and *I* is proportional or noninear. The proof that Eqs. (20) and (22) are identical is given in Section (16).

- Equations (21) and (23) are *identical*. Both equations are applied whether the relationship between q and ΔT is proportional or nonlinear.

  The particular advantage of Eq. (23) is that, if the relationship between q and ΔT is nonlinear, problems are *much* easier to solve using Eq. (23). The proof that Eqs. (21) and (23) are identical is given in Section (17).

16. The proof that Eqs. (20) and (22) are *identical*, and how they compare.

In Eq. (20), substitute E_n/I_n for R_n.

$$R_{parallel} = 1/(1/R_2 + 1/R_3 + 1/R_4) \quad (20)$$

$$(E_{parallel}/I_{parallel}) = 1/(I_2/E_2 + I_3/E_3 + I_4/E_4) \quad (24)$$

Since R_2, R_3, and R_4 are in parallel,

$$E_2 = E_3 = E_4 = E_{parallel} = E \quad (25)$$

$$(E/I_{parallel}) = 1/(I_2/E + I_3/E + I_4/E) \quad (26)$$

$$(E/I_{parallel}) = 1/((I_2 + I_3 + I_4)/E) \quad (27)$$

$$(E/I_{parallel}) = E/(I_2 + I_3 + I_4) \quad (28)$$

$$I_{parallel} = I_2 + I_3 + I_4 \quad (22)$$

Q.E.D.

16.1 Comparison of Eqs. (20) and (22).

- Equation (20) is used to solve *only* those problems in which E is proportional to I. Equation (22) is used to solve problems whether E *is or is not* proportional to I.
- If Eq. (20) is used, solutions involve *three* parameters—E, I, and R. If Eq. (22) is used, solutions involve *two* parameters—E and I.
- Equation (20) is *opaque* because it conceals that it means "the total current is the sum of the individual currents". Equation (22) is *transparent* because its meaning is readily apparent.

17. The proof that Eqs. (21) and (23) are *identical*, and how they compare.

Fluids 1 and 2 are separated by a flat plate. Therefore Eq. (29) applies.

$$q_1 = q_{wall} = q_2 = q \quad (29)$$

In Eq. (21), substitute $q/\Delta T_{total}$ for U, $q/\Delta T_1$ for h_1, $q/\Delta T_{wall}$ for k_{wall}/t_{wall}, and $q/\Delta T_2$ for h_2.

$$U = 1/(1/h_1 + t_{wall}/k_{wall} + 1/h_2) \quad (21)$$

$$q/\Delta T_{total} = 1/(\Delta T_1/q + \Delta T_{wall}/q + \Delta T_2/q) \quad (30)$$

$$q/\Delta T_{total} = q/(\Delta T_1 + \Delta T_{wall} + \Delta T_2) \quad (31)$$

$$\Delta T_{total} = \Delta T_1 + \Delta T_{wall} + \Delta T_2 \quad (23)$$

Q.E.D.

17.1 Comparison of Eqs. (21) and (23)

- In modern heat transfer engineering, Eq. (21) is used to solve problems whether q is or is not proportional to ΔT.
- If q is *not* proportional to ΔT, $q/\Delta T$ is a *variable*, and problem solutions based on Eq. (21) involve *three* thermal *variables*—q, ΔT, and $q/\Delta T$.
- Problem solutions based on Eq. (23) *always* involve only *two* thermal variables—q and ΔT.
- Equation (21) is *opaque* because it conceals that it means "the total temperature difference is the sum of the individual temperature differences".
- Equation (23) is *transparent* because its meaning is readily apparent.

18. Problem 1—An elastic behavior problem that demonstrates the application of $\sigma = f\{\varepsilon\}$.

Solve Problem 1 using $\sigma = f\{\varepsilon\}$ instead of $\sigma = E\varepsilon$.

18.1 Problem 1 Statement

Given the bar described in Section 18.3, determine the following:

- Axial load that would increase the length of the given bar by .05 centimeters.
- Stress that would result in the given bar.
- Strain that would result in each region of the given bar.

18.2 Problem 1 Nomenclature

σ is the numerical value of stress.

ε is the numerical value of strain.

The dimension units that underlie the parameter symbols are kilograms and centimeters.

18.3 Problem 1 Given
The bar consists of two regions of different materials. Region 1 is 65 centimeters long. Region 2 is 50 centimeters long. Equations (32) and (33) describe the material behavior in each region. The cross-section of the bar is everywhere 9 square centimeters.

$$\sigma_1 = 1.7x10^6\ \varepsilon_1 \text{ for } \varepsilon_1 < .002 \tag{32}$$

$$\sigma_2 = 1.2x10^6\ \varepsilon_2 \text{ for } \varepsilon_2 < .002 \tag{33}$$

18.4 Problem 1 Analysis
Equations (34) and (35) result from the given information.

$$\sigma_1 = \sigma_2 = \sigma \tag{34}$$

$$65\varepsilon_1 + 50\varepsilon_2 = .05 \tag{35}$$

Substitute the given information in Eq. (35), and use Eq. (34).

$$65\sigma/(1.7x10^6) + 50\sigma/(1.2x10^6) = .05 \tag{36}$$

18.5 Problem 1 Answer
Solution of Eq. (36) gives a stress of 626 kilograms per square centimeter. Since the bar cross-section is 9 square centimeters, the load is 5,630 kilograms.

Eq. (32) indicates the strain in Region 1 is .00037. Eq. (33) indicates the strain in Region 2 is .00052.

19 Inelastic problems that demonstrate the application of $\sigma = f\{\varepsilon\}$.
In Problem 1, if the exponent of ε_1 in Eq. (32) is 1.2 instead of 1.0, the first term in Equation (36) is $65(\sigma/(1.7x10^6))^{.833}$ instead of $65\sigma/(1.7x10^6)$, and the solution of Equation (36) is $\sigma = 238$.

The answer to Problem 1 is: the load is 2142 kilograms; the stress is 238 kilograms per square centimeter; the strain in Region 1 is .000616; the strain in Region 2 is .000198.

In Problem 1, if the material behavior in Region 1 is described by a nonlinear stress/strain chart in place of Eq. (32), the problem is solved by the following graphical solution of Eq. (35):

- Use the given stress/strain chart to prepare a chart of $65\varepsilon_1$ vs. σ.
- Use Eq. (33) to determine $50\varepsilon_2$ vs σ, and plot it on the prepared chart.
- On the prepared chart, plot $(65\varepsilon_1 + 50\varepsilon_2)$ vs. σ.
- The stress in the bar is the chart stress at the point (or points) at which $(65\varepsilon_1 + 50\varepsilon_2)$ equals 0.05.
- If the given stress/strain chart is highly nonlinear, there may be more than one point at which $(65\varepsilon_1 + 50\varepsilon_2)$ equals 0.05. In that event, stability analysis is required in order to determine a unique solution. Stability analysis is beyond the scope of this monograph.

The principal disadvantage of Young's law and elastic modulus is that they do *not* apply if the behavior is inelastic.

For example, if the exponent of ε_1 in Eq. (32) is 1.2 instead of 1.0, the behavior in Region 1 is inelastic. Consequently Young's law and elastic modulus do *not* apply in Region 1, and Problem 1 is *not* solved using Young's law. Other methodology is used.

The equation $\sigma = f\{\varepsilon\}$ is *always* obeyed—ie it is obeyed in elastic regions, *and* in inelastic regions. Therefore the equation $\sigma = f\{\varepsilon\}$ is appropriately named "the *law* of material behavior".

20. Problem 2—An example that demonstrates the application of $E = f\{I\}$ to problems that concern conductors that obey Ohm's law.
Solve Problem 2 using $E = f\{I\}$ instead of $E = IR$.

20.1 Problem 2 Statement
Determine E and I for each conductor in the circuit described in Section 20.3.

20.2 Problem 2 Nomenclature
E is the numerical value of electromotive force.

I is the numerical value of electric current.

The dimension units that underlie the parameter symbols are volts and amperes.

20.3 Problem 2 Given

The circuit contains Conductor 1 in series with parallel Conductors 2, 3, and 4.

Eqs. (37) through (40) describe the electrical behavior of each conductor.

$$E_1 = 6I_1 \quad (37)$$

$$E_2 = 22I_2 \quad (38)$$

$$E_3 = 15\ I_3 \quad (39)$$

$$E_4 = 32\ I_4 \quad (40)$$

The total emf is 100 volts.

20.4 Problem 2 Analysis

$$E_2 = E_3 = E_4 = E_{2,3,4} \quad (41)$$

$$I_2 + I_3 + I_4 \equiv I_{2+3+4} = .045E_{2,3,4} + .067E_{2,3,4} + 029E_{2,3,4} = I_1 \quad (42)$$

$$I_{2+3+4} = .1434E_{2,3,4} \quad (43)$$

$$E_{2,3,4} = 6.97I_{2+3+4} = 6.97\ I_1 \quad (44)$$

$$E_1 + E_{2,3,4} = 100 \quad (45)$$

Combine Eqs. (37), (44), and (45).

$$6I_1 + 6.97I_1 = 100 \quad (46)$$

$$I_1 = 7.71 \quad (47)$$

$$E_{2,3,4} = E_{total} - E_1 = 100 - 6x7.71 = 53.7 \quad (48)$$

$$I_2 = E_{2,3,4}/22 = 53.7/22 = 2.44 \quad (49)$$

$$I_3 = 53.7/15 = 3.58 \quad (50)$$

$$I_4 = 53.7/32 = 1.68 \quad (51)$$

20.5 Problem 2 Answer

$E_1 = 46.3$, $I_1 = 7.71$

$E_2 = 53.7$, $I_2 = 2.44$

$E_3 = 53.7$, $I_3 = 3.58$

$E_4 = 53.7$, $I_4 = 1.68$

21. The application of *E* = f*{I}* to problems that concern conductors that do *not* obey Ohm's law.

In Problem 2, if the exponent of I_1 in Eq. (37) is 1.2 instead of 1.0, the first term in Eq. (46) is $(6I_1)^{1.2}$ instead of $6I_1$, and the solution of Eq. (46) is $I_1 = 6.38$. Also $E_1 = 55.5$, $E_{2,3,4} = 44.5$, $I_2 = 2.02$, $I_3 = 2.97$, $I_4 = 1.39$.

In Problem 2, if the electrical behavior of Conductor 1 is described by a nonlinear E_1 vs I_1 chart in place of Eq. (37), Problem 2 is solved by the following graphical solution of Eq. (46):

- On the chart of E_1 vs $\underline{I_1}$, plot $E_{2,3,4}$ vs. I_1.
- On the chart of E_1 vs $\underline{I_1}$, plot $(E_{2,3,4} + E_1)$ vs. I_1. The value of I_1 is the chart value of I_1 at the point (or points) where $(E_{2,3,4} + E_1)$ equals 100.
- If the E_1 vs I_1 chart is highly nonlinear, there may be more than one point at which $(E_{2,3,4} + E_1)$ equals 100. In that event, stability analysis is required. Stability analysis is beyond the scope of this article.

The principal disadvantage of Ohm's law and electrical resistance is that they apply *only* to conductors that obey Ohm's law.

If the exponent of I_1 in Eq. (37) is 1.2 instead of 1.0, Conductor 1 does *not* obey Ohm's law. Consequently Problem 2 is *not* solved using Ohm's law and electrical resistance. Other methodology is used.

The equation $E = f\{I\}$ is *always* obeyed—ie it is obeyed whether conductors *do or do not* obey Ohm's law. Therefore the equation $E = f\{I\}$ is appropriately named "the *law* of resistive electrical behavior".

22. An example problem that demonstrates the application of *q* = f*{ΔT}* to problems in which boundary layers exhibit proportional thermal behavior

Solve Problem 3 using $q = f\{\Delta T\}$ instead of $q = h\Delta T$.

22.1 Problem 3 Statement

In the system described in Section 22.3, determine the following:

- Heat flux from Fluid 1 to Fluid 2.
- Temperature at each fluid/wall interface.

22.2 Problem 3 Nomenclature

q numerical value of heat flux.

T numerical value of temperature.

ΔT_{BL1} numerical value of temperature difference across the Fluid 1 boundary layer.

ΔT_{BL2} numerical value of temperature difference across the Fluid 2 boundary layer.

ΔT_{wall} numerical value of temperature difference across the wall.

The dimension units that underlie the symbols are watts, degrees centigrade, and meters.

22.3 Problem 3 Given

Fluids 1 and 2 are separated by a large flat wall that is 0.006 meters thick.

Eqs. (52) to (54) describe the thermal behavior of the fluid boundary layers and the wall.

The Fluid 1 temperature is 255 C. The Fluid 2 temperature is 147 C.

$$q_{BL1} = 620\ \Delta T_{BL1} \tag{52}$$

$$q_{BL2} = 470\ \Delta T_{BL2} \tag{53}$$

$$q_{wall} = 1250\ \Delta T_{wall} \tag{54}$$

22.4 Problem 3 Analysis

Substitute the given information in Eq. (23).

$$\Delta T_{total} = \Delta T_1 + \Delta T_{wall} + \Delta T_2 \tag{23}$$

$$\Delta T_{total} = \Delta T_{BL1} + \Delta T_{wall} + \Delta T_{BL2} = T_1 - T_2 = 255 - 147 = 108 \tag{55}$$

Because the wall is flat, $q_{BL1} = q_{wall} = q_{BL2} = q$.

Substitute the given information in Eq. (55).

$$\Delta T_{total} = q/620 + q/1250 + q/470 = 108 \tag{56}$$

$$\Delta T_{total} = .00454q = 108 \tag{57}$$

$$q = 23{,}800 \tag{58}$$

$$T_{interface\ 1} = T_1 - \Delta T_{BL1} = 255 - 23{,}800/620 = 217 \tag{59}$$

$$T_{interface\ 2} = 147 + \Delta T_{BL2} = 147 + 23{,}800/470 = 198 \tag{60}$$

22.5 Problem 3 Answer

- The heat flux from Fluid 1 to Fluid 2 is 23,800 watts per square meter.
- The temperature of Interface 1 is 217 C.
- The temperature of Interface 2 is 198 C.

23. The solution of heat transfer problems in which boundary layers exhibit nonlinear thermal behavior.

Heat transfer engineering does *not* include a second methodology that is used if a problem concerns nonlinear thermal behavior. Heat transfer problems are solved using $q = h\,\Delta T$ whether boundary layers exhibit proportional behavior or nonlinear behavior.

Until sometime near the beginning of the twentieth century, $q = h\Delta T$ was a *law* that indicated the relationship between q and ΔT is *always* proportional, and h is *always* a *constant* of proportionality.

$q = h\Delta T$ ceased to be a law when it was determined that boundary layers do *not* always exhibit proportional thermal behavior. If the heat transfer is by natural convection or condensation or boiling, boundary layers exhibit *nonlinear* thermal behavior.

In order to make the law apply to both proportional and nonlinear thermal behavior, the law was transformed to what some texts refer to as "the defining equation for *h*".

In other words, $q = h\Delta T$ ceased to be an equation, and became the definition $h \equiv q/\Delta T$. And h ceased to be a proportionality constant, and became *either* a proportionality constant or a *variable*, depending on whether or not q is proportional to ΔT.

When the law was transformed from equation to definition, $q = h\Delta T$ should have been replaced by $h \equiv q/\Delta T$, and the designation "law" should have been discontinued. However, both $q = h\Delta T$ and the designation "law" were retained.

The principal disadvantage of using heat transfer coefficient $q/\Delta T$ is that it *moderately* complicates the solution of problems that concern natural convection or condensation, and *greatly* complicates the solution of problems that concern boiling. The complication results from the fact that, if the relationship between q and ΔT is nonlinear, $q/\Delta T$ is a *variable* rather than a constant.

The principal advantage of using $q = \mathrm{f}\{\Delta T\}$ is that it works well with *both* proportional and nonlinear thermal behavior because q and ΔT are kept *separate*, thereby *eliminating* the variable $q/\Delta T$ in problems that concern nonlinear thermal behavior.

The equation $q = \mathrm{f}\{\Delta T\}$ is a *law* because it is *always* obeyed. Therefore the equation $q = \mathrm{f}\{\Delta T\}$ is appropriately named "the *law* of convective heat transfer behavior".

24. How the abandonment of laws, and contrived parameters in the laws, impact engineering curricula.

The abandonment of laws such as Young's law and Ohm's law, and contrived parameters such as elastic modulus and electrical resistance, has the following impact on engineering curricula:

- Students learn to think in terms of *real* parameters rather than *contrived* parameters. *Real* parameters such as stress and strain and electromotive force and electric current, rather than *contrived* parameters such as modulus σ/ε and electrical resistance E/I.
- Students learn how to solve engineering problems with the primary parameters *separated* (the methodology learned in mathematics) rather than *combined* in contrived parameters such as elastic modulus σ/ε and electrical resistance E/I.
- *Throughout* their engineering studies, students learn to think and to solve problems in a way that applies to *all* forms of behavior.
- Students do *not* begin their studies by learning to think and to solve problems in a way that applies to proportional behavior, then learning a different way to think and to solve problems in a way that applies to nonlinear behavior.

25. CONCLUSIONS

Modern engineering methodology should be modified in order to correct for the impact of Fourier's error of induction. The required modifications are:

- Parameter symbols in equations represent numerical value but *not* dimension.
- Dimension units that underlie parameter symbols in quantitative equations are specified in accompanying nomenclatures.
- Parametric equations contain *only* numbers, and consequently are *inherently* homogeneous.
- All laws that concern two parameters are analogs of $y = \mathrm{f}\{x\}$.
- Equation (64) replaces Young's law, and is named "the law of material behavior".

$$\sigma = \mathrm{f}\{\varepsilon\} \tag{64}$$

- Equation (65) replaces Ohm's law, and is named "the law of resistive electrical behavior".

$$E = \mathrm{f}\{I\} \tag{65}$$

- Equation (66) replaces Fourier's law of convective heat transfer, and is named "the law of convective heat transfer behavior".

$$q = \mathrm{f}\{\Delta T\} \tag{66}$$

- There are *no* contrived parameters such as elastic modulus σ/ε, electrical resistance E/I, and heat transfer coefficient $q/\Delta T$.

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